The time of populations: Reconstruction and Dynamics

Noël BONNEUIL

Institut national des Ãľtudes démographiques,

133, bld Davout, 75980, Paris cedex 20, France,

and

École des hautes Ãľtudes en sciences sociales,

54, bld Raspail, 75006, Paris, France

bonneuil@ined.fr

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1 The Time of Populations

The unexpected Part of population studies concern stable population models, convergence toward equilibria, cycles, asymptotic distributions. Yet, human populations, because of history, call for a different view of time. For example, fertility time-series display sudden changes, breaks, jumps from one regime to another, temporary stagnation, unexpected... This variability of time signals raises the question of the viability of underlying systems: how, in our advanced economies, do we manage the fits and starts of the age structure, such as the massive arrival of schooled children then students than job seekers, then now pensioners? How, in traditional populations, can a society pass through epochs of wars, plagues, and dearth? How does family limitation appear and diffuse in territory and society, as it was the case in France from mid-eighteenth century throughout the nineteenth century?

Re-founding concepts linked to the time of humans The time of populations covers the non linearity of processes, where small effects can bring about large changes, a context of uncertainty, and the possibility of action and human agency. The concepts of trajectory and equilibrium are no longer adequate to tell on causality. This is why they ought to be superseded respectively by the concepts of attainable set and by the maximal set of states from which there exists at least one solution allowing the system to perpetuate itself, and, as a complement, the set of states from which the system is doomed to failure. The concept of optimal strategy familiar to economists or biologists is replaced by the concept of viable strategy, and optimal gain or utility by capture-viability of achievements *ex ante* or *ex post*. Forecasting is replaced by how to obtain the maintenance within a set of constraints or by the capturability of a target set. Priority is given to transient dynamics, allying temporary permanence and changes in social forms, instead of asymptotic considerations (among which stable population models, ergodic theorems, demo-economic or biological cycles).

When I was introduced to viability theory in 1992, this theory was a pure abstraction. Its "illustrations" were pure school exercises. I was longing at mathematics of time adapted to my own idea of time in historical social situations. Viability theory helps us answer questions outside the frame of probability theory, help us deal with the time of populations outside equilibria and without the constructed concept of trajectory. Typically, when I was invited to comment on anthropologist Fredrik Barth's work, I was supposed to talk about game theory. However, reading it made it clear that, behind the metaphor of this static theory, Fredrik Barth was talking on the maintenance of nomadic populations or on the dynamic game among fishermen, that he had well understood the themes at stake in viability theory.

Returning to data, always I took benefit of original data which have required a specific and innovative correction treatment (as with parish registers[1] or the Statistique GÃl'nÃl'rale de la France,[2]) organization (as in the genealogical chaining of the TRA families), reconstruction (I have suggested several reconstruction methods), or import (as with DNA data). I have gradually constituted a global and minute fresco of the time of populations. I pioneered population reconstruction methods (the "trend projection" for parish registers, the reconstruction of migrations and under-registration in nineteenth century France, the construction of population paths the closest to a stable one in paleo-demography, the reconstruction of populations with missing data); I suggested an original

[1]Bonneuil N. (1998), Traitement des donnÃl'es manquantes dans les sÃl'ries issues des registres paroissiaux, *Population*, no spÃl'cial Histoire 1-2, 249-272.

[2]Bonneuil N. (1989) Cohérence démographique des tableaux de la Statistique Générale de la France, 1801-1906, *Population* 4-5, 809-840.

use of factor analysis for qualitative sequences; I introduced age in marker association segregation analysis in genetic epidemiology. The various fields I have addressed led me to new questions, be it for concepts or mathematical techniques. Raising the question of time in demography, and no longer the question of the kinematic consequences of established conditions, as in Lotka or Easterlin, leads to take the risk of inter-disciplinarity.

1.1

1.1.1 Fertility Fluctuations in the Old Demographic Regime

Territorial Regulation in *Pays de Caux*, 1588-1700 The French seventieth century constitutes an exceptional field to observe populations experiencing drastic conditions of mortality and subsistence. The plague occurred frequently, let alone war, dysentery, or dearth. Parish series (baptisms, marriages, burials) look very irregular. To exploit these data, certain historians emphasize the peaks of deaths, which they call "mortality crises." Others focus on the study of short-term fluctuations with econometric models.[3]

It is more difficult to reconstruct populations of the past, to highlight the processes of mortality and fertility. I considered the case study of the population of Pays de Caux, 1588-1700. Censuses do not exist for such traditional populations, and I suggested an original method based on the sole time series of parish registers to obtain fertility and life expectancy time series over these 113 years of old demographic regime.

Their very irregular appearance forbids one to understand the underlying dynamics in sticking to traditional econometrics. Yet, replacing these trajectories in state space restores the discontinuities, the accelerations or the slowing down movements. This operation delivers the anatomy of crises in a clinical manner or the contextual shocks. A model

[3]Bonneuil (1991) Temporalités en démographie historique, *Histoire et Mesure* VI-1/2, 137-148.

weaving together constraints of ground exploitation, nuptiality, and fertility of the couples to the ups and downs of mortality turns out to be a good explanatory candidate, able not only to mimic all reconstructed series, but also to retrace the series of observed numbers of marriages, a series independent so far of the reconstruction and the simulation.

The fertility of this population goes back and forth between two levels, appearing as two attractors in the phase space. After a long enough period without mortality crisis, married women are old enough, and the mean fertility is low. Brides are also old enough, and do not raise the mean fertility. Most young women must wait before marrying for a farm to be freed by the decease of its landowners. When a mortality crisis occurs, as in 1639-40, marriable people are numerous enough to replace the deceased landowners, and already old enough to maintain the fertility level still at a low level. When mortality crises repeat themselves, then the population of marriable people is depleted, and brides are chosen younger and younger, contributing to raise fertility to a high level, where it is maintained by the high frequency of mortality peaks and its subsequent renewal of landowners and their wives. When mortality crises calm down, fertility decreases and returns to the low level at the same rhythm that married women age and their fertility decreases.

At last, a model adapted to the empirical data of the Pays de Caux allows us to deepen the consequences of such a relationship between populations and resources. Simulating this population model and varying the probability of occurrence of mortality crises, I showed that there exists a bifurcation value, or a value beyond and below which the system works in different regimes: beyond eight crises per century, fertility always remains high, maintained by high mortality, and the population, although fertile, is threatened to be annihilated if mortality eventually goes out of reach of fertility; between 1 and 8 mortality crises per century, we observe a two-level regime, and we find again the empirical case of Pays de Caux; in the absence of crisis, fertility is confined in the low level regime, corresponding to late age at marriage. This case study for the first time allowed the validation of the territoriality model with genuine historical data. It highlights the topological properties associated with the historical fluctuations of mortality and the responses of the age-pyramid.[4]

Viability in Population and Environment (Malthus-Boserup) This case study raises the question of the interactions between population and environment. After Malthus denounced population growth as a cause of impoverishment, Boserup sustained that a large population has a positive effect on technological innovation. Yet, certain empirical observations go in the direction of Boserup, others do not.

I suggested to revise the theoretical framework of this debate in giving a central place to inertia inherent in technological change, and in considering innovation no longer as a mechanistic process, but as a set-valued function encompassing possible futures.[5] This allows us to find the ideas of Boserup without postulating them, and to re-establish contingency and indeterminacy in agents' behavior, instead of thinking them as submitted to mechanistic forces.

In representing the state space of the Malthus-Boserup system under constraints, I showed how the Boserupian hypothesis enters into a larger view, where myriad Boserupian dynamics coexist with other non-Boserupian ones. With time passing and population growing, the Malthusian equilibrium, characterized by low income, gets every day closer, with the space of attainable states shrinking, constituting an ever pressing incentive to do something, to innovate and rescue the situation. But this is not compulsory, as it is

[4]Bonneuil N. (1990) Turbulent Dynamics in a 17th century population, *Mathematical Population Studies* 2, 289-311.

[5]writing a differential inclusion $x' \in F(x)$.

observed empirically and as it is retraced by the qualitative properties of the dynamical system.[6]

Viability among Nomads and Fishermen (from anthropologist Fredrik Barth)

Anthropologist Fredrik Barth (1981) described another traditional society, that of Basseri nomads. He showed how this society was organized so as to perpetuate itself, and he identified the processes with which the unity of the camp was maintained. When Barth resorts to the concept of equilibrium and static games to explain the history of the group, I showed[7] that he actually talked of dynamical games and viability out of equilibria. Equilibrium on the contrary means starvation, and the problem of nomads is specifically to maintain themselves far from this equilibrium as long as their resources and their demography allow them to do so. To manage this, the group uses 'strategies' (in the sense of game theory): successively the choice of the site where to settle temporarily (when to leave the camp and where to go?), the reduction of individual consumption in situation of food shortage, and finally the sedentarization of the poorer, going to sell their work force in farms, so as to relieve livestock from human demand. If these regulations fail, mortality increases until population is adjusted to resources. In this story, nomads take decisions under the pressure of present and future constraints, they restlessly alter they own history, their 'trajectory' in the space of possible states. They cannot contemplate their future in terms of trajectory, but rather in terms of sets of attainable states, and the head of the group is specifically the one in charge of taking viable decisions, doing his best

[6] Bonneuil N. (1994) Malthus, Boserup and Population Viability, *Mathematical Population Studies* 4(5), 107-119.

[7] Bonneuil N. (1997) Games, Equilibria, and Population Regulation under Viability Constraints: an interpretation of the work of the anthropologist Fredrik Barth, *Population English Selection* special issue New Methods in Demography, 151-179.

less to find the optimal choice of which he has no knowledge than to simply avoid failure; technically speaking, these decisions are those which lead the group in the interior of the set of survival constraints of the group (with the additional difficulty that the interactions between humans and livestock are not linear). In this concrete example, decision-making in uncertainty (stemming from unpredictability inherent in availability of land, in weather forecast, in natural misfortunes, ...) is mandatory. Instead of a chronological time, we have a social time, where a finite number of controls makes possible a myriad different stories.

This traditional population has then invented original preventive checks, exempting couples to resort to delay at marriage or to family limitation. It perpetually invents its own history, according to its resources and its development.

In another study of Norwegian fishermen, Barth showed that skippers must continually choose between following the other vessels or striking out on their own, searching for new herring shoals. The strategy changes then all the time with respect to the movements of other vessels. I showed that it was in fact a dynamical problem, in delineating the *capture domain* in the phase space constituted by the capital (ship and crew) in time t, by the volume of catch, and by the probability at t to find the best catch in taking risk. This is no longer a question of what other actors will do, but to determine the set of states from which a skipper has the possibility to avoid ruin, and which strategies, with respect to the system, will allow him to do so. How will a skipper effectively react, in a more or less clever manner, produces a story; the capture domain and its associated states of viable strategies discriminate successes and failures among all possible stories. As in this concrete case, the modern theory of dynamic games extends the theory of games, and allows us to overcome the limitations inherent in the static case of game theory.

1.1.2 Diffusion of Behaviors in the Demographic Transition

The French transition 1806-1906 Between fertility fluctuations at low levels since the beginning of the twentieth century and those at high levels in the seventeenth century, the demographic transition raises the question of the change of fertility behaviors in time and space. I explored this question through the data of the *Statistique Générale de la France* from 1806 to 1906 by *départements*, the French administrative subdivisions counting up to 90 for the metropolis in the nineteenth century.

The frequent censuses (every five years) are detailed at the level of *département*, and the statistics of deaths by age were yearly from 1851 onward. These sources are, as many other statistics, flawed with counting errors, under-recording, or declaration defects. The question of the quality of the data is often neglected, but it is inherent to demographic statistics, and this is a difficulty which I addressed. The taking into account of under-recording wiped up the myth of the baby-boom of the 1870's, and situated the French fertility decline, which, although still pioneer in Europe, was not as marginal as it was claimed (with data not corrected of under-registration).

I suggested an original reconstruction,[8] capable of correcting the SGF data from, notably, the data of deaths by age. I obtained a new panorama of demographic forces (fertility, mortality, net migration by age). Incorporating explanatory variables such as urbanization or female education, the transition appears as a dynamic system in space and time. Appropriate econometric techniques (cluster analysis, factor analysis, simultaneous regressions, co-integration) allow its study. Notably, urban hierarchy, which was concealed in the absence of reconstruction of urban data in the book by van de Walle (1974), reveals itself as a key feature of the transition. The process is double-sided: at the beginning

[8] Bonneuil N. (1997) *Transformation of the French demographic landscape*. Oxford : Clarendon Press, 224 pages.

of the transition, fertility behavior adjusts to a varying harsh environment, then, once launched, the decline goes on, through an innovation wave travelling through the territory from diffusing centers.

A problem, often encountered in the analysis of historical data, is the difficulty in overcoming missing or flawed data. Lotka-McKendrick discrete demographic model, including migration, is combined with stochastic optimization to fit available censuses and vital statistics series to reconstruct missing population data, in the presence of one or two censuses. Simulations help to calibrate the method and determine error weights associated with each data series. An empirical case study is made using data from an administrative subdivision in southern Russia for the period 1863-1916.[9]

The reconstruction of the population by marital status from imperfect statistics is obtained as solution of another minimization program in large dimension. The correspondence of ages between brides and grooms at each month of the year between 1867 and 1916 also results from a stochastic optimization, avoiding the introduction of ad hoc marriage functions.[10]

The influence of economics in the demographic transition From the theoretical point of view, I examined the relationship between fertility, mortality, and husband-wife productivity ratio through the unilateral gift equilibrium.[11] Bergstrom (AER, 2007) showed why fertility during the demographic transition has no reason to be correlated with productivity, but that it should be so with the husband-wife productivity ratio. He used

[9]Bonneuil N. and Fursa, E. (2011) Optimal Population Path Fitting for Flawed Vital Statistics and Censuses, *Journal of Optimization Theory and Applications* 148-2, 301-317.

[10]N. Bonneuil et E. Fursa, (2012) Optimal Marriage Fitting for Imperfect Statistics, *Journal of Optimization Theory and Applications*, 153: 532-545.

[11]Bonneuil N. (2010) Family Regulation as a Moving Target in the Demographic Transition, *Mathematical Social Sciences* (Elsevier) 59, 239-248.

a static framework at equilibrium. I showed firstly that re-introducing mortality in the unilateral gift equilibrium gives mortality the leading role of the transition, as observed everywhere. Secondly, I showed that introducing learning and inertia changes the view of a shifting equilibrium to a view of a system pursuing a moving target. The demographic transition appears then as driven by mortality and economics although correlations of observed fertility with productivity and with husband-wife productivity ratio is null, a result common to acutely documented studies of the transition.

The influence of religion in the demographic transition The quality of the statistical archives of the Don Army Territory for the period 1867-1916 offers a unique opportunity to compare the denominations present (Orthodox, Old Believers and Coreligionists, Catholics, Lutherans, Jews, Armenian-Gregorians, Lamaist Buddhists, and Muslims) in a context of urbanization, secularization, and industrialisation. The observance of religious interdicts varied between and within denominations depending on levels of urbanization and the district considered. These contrasts reveal the segmentation of Don society, the constraints imposed by agrarian imperatives and military obligations, the differences in the economy and between town and country, and the advance of secularisation or persistence of tradition.

In the countryside, the constraints of agricultural work combined with religious rules and superstitions and the rigours of climate, to impose seasonal patterns of marriage. The timing and intensity of the agricultural cycle also varied with the district of residence and type of economic activity (nomadic, industrial, horse-rearing, or grain-growing) of the denominational groups. City-dwelling and industrial work freed people, partly or entirely, from agricultural work, as shown by the comparison between rural and urban Lutherans or between rural and urban Orthodox. The question was then addressed of whether city-dwelling favoured secularisation. Different denominations reacted in different ways when no longer bound by the imperatives of the agrarian calendar: among urban Orthodox and Old Believers respect for the religious calendar was undiminished; in contrast, urban Armenian-Gregorians, Catholics, Lutherans, Jews, emancipated themselves from their respective calendars. Muslims constitute a unique denomination from the viewpoint of marriage seasonality, in that for them marriage was permitted in all months.

Following up seasonal components of marriages over time showed no relaxation of calendar interdicts among Orthodox and Old Believers/Coreligionists. No marriages were registered for these groups in March or December before 1916. So the change observed for urban Orthodox and Old Believers/Coreligionists (the slow rise of July and September, the slow decline of November, the smaller dispersion compared to country-dwellers) reflects the partial liberation from agricultural tasks more than a hypothetical secularization. As expected, village landowners who defended the values of the peasant family, remained attached to a more restrictive, more traditional matrimonial system. In contrast, the seasonal components of marriage revealed the pace with which secularization gained ground among Catholics, Lutherans, Jews, and above all urban Armenian-Gregorians, reflected in the relaxation of religious obedience.

The compliance with or relaxation of the seasonal pattern of marriage reveals the cleavage produced in Don society by Tsarism and Revolution, but also that between a rural economy trading mainly in grain, horses, and fish, and a growing industrial economyâĂŤnotably with the working of the Donetz minesâĂŤrun by Lutherans, Armenian-Gregorians, and Jews.

The abandonment of an agricultural way of life appears a necessary prelude to secularization, but it is not sufficient, as the case of the Orthodox and Old Believers shows. The set of changes that gradually redefined nuptiality in this region of Russia sharpened the existing divisions both between denominational groups and also within them, exposing the rupture between traditionalists and modernists. The seasonality of marriages thus functions as a marker for what would become a major line of cleavage in Don society during and after the 1917 Revolution.

1.2 Social Mobility in nineteenth century France

Beside this work on French and Egyptian censuses, I contributed to exploit a nominative survey on nineteenth century, the so-called TRA survey, gathering around 45,000 marriage certificates belonging to 3000 genealogies followed up in time. The TRA data are convenient to the study of social mobility. Scholars usually build transition matrices, where fathers and sons are classified with respect to pre-established categories. Yet, occupation at that time has a variable meaning according to epoch or location. I have then suggested to build a continuous score associated to an occupation, net the effects of time or location. Fathers and sons or mothers and daughters become then comparable without the need of pre-defined categories. The transformation father-son is defined as a correspondence in the mathematical sense, which is to be characterized by segmenting with econometric models.[12] With P. A. Rosental, we could then structure the geography and the French social space with respect to social mobility, taking into account education, occupation, residence, parents' mortality, and time.

1.3 Familial Components of First Migrations after Marriage

The genealogical chaining of marriage certificates in the 3000 families survey opens the unique possibility to study the familial components of migrations. The familial interde-

[12]Bonneuil N. and P.A. Rosental (1999) Changing social mobility in 19th century France, *Historical Methods* Spring 32(2), 53-73.

pendency of migration behaviors forbids to consider individuals as independent from each other. On the contrary, I have suggested a multi-level multi-process event-history analysis, where it is at last possible to test the role of familial 'network'.[13] We confirmed the presence of two processes of migration, short- and long-distance. We highlighted the importance of the occupations of parents and parents-in-law in the occurrence of migration, of elder brother's and sister's residence and occupations, of the influence of primo- and ultimo-geniture, and the ability to read and write which creates a selection effect of migrants to cities.

1.4 Event-history Analysis in Contraception, in the Labor market

Event-history analysis and life cycle is an important aspect of demography. In a family planning perspective, with Margarita Medina (CED Barcelona) I studied the transition of contraception in Colombia from multilevel multiprocess event history analysis[14] where we highlighted the ambivalence of young women toward contraception, shared between the need of self-assessment of the couple and modernity. In labor economics, I analyzed the 1990 fertility survey to describe the determinants of transitions on the female labor marke.t[15] In personnel economics, I decomposed the fluctuations of total wages in a firm in the fluctuations due to the structure of ranks, to technicity change, and to seniority progression.[16]

[13]Bonneuil N., A. BringÃľ and P.A. Rosental (2008) Familial Components of First Migrations after Marriage in nineteenth century France, *Social History* 33-1, 36-59.

[14]N. Bonneuil and M. Medina (2010) The transition of contraception in Colombia 1950-1994, *Desarollo y Sociedad*.

[15]Grimm M. and N. Bonneuil (2001) Labor market participation of French women over the life-cycle, 1935-1990, *European Journal of Population* 17(3), 235-260.

[16]Bonneuil N. (1989) Démographie du personnel : effet de structure sur l'évolution de la masse salariale dans une entreprise Ãă forte mobilité, *Population* 6, 1101-1120.

With Chouaa Dassouki, I also investigated the demographic transition in Egypt from 1960 to 1996, on one hand at the spatial scale of *Muhafazas* (governorates), most of them divided into rural and urban, or 38 subdivisions covering the territory,[17] on the other hand at the scale of *qaryas, medinas, shiyakhats, qisms*, or 4905 administrative subdivisions.[18] Thanks to its geometry and its contrasted economic and political history, the case of Egypt is highlighting. Linking time series of fertility, mortality, and net migration with times series of education, civil status, and employment in various sectors impinges on difficulties specific to the statistical treatment of the demographic transition, where numerous variables vary jointly and are auto-correlated in space and time. The model tackles these difficulties, and allows the testing of diffusion from chief town of governorate, from big cities (Cairo, Alexandria, Suez, Port Saïd) or from closest neighbors. One noteworthy result is that, after the economic, health, education, and family planning effects, there remains an important spatial diffusion of behaviors, contributing to a holistic portrayal of the transition. Another result is that the rapidity of change in the seventies brought about a strong heterogeneity within a same governorate: the transition does not alter behaviors uniformly, but causes disparities which take time to vanish. The short baby-boom in 1974-85 pauses these disparities for a while, above all in Upper Egypt, because fertility rises abruptly in the subdivisions which are advanced in the transition, under the impulse of an economic boom and a political seizure.

[17] Bonneuil N. and C. Dassouki (2007) Economics, Geography, Family Planning and Rapidity of Change in the Demographic Transition: the Case of the Egyptian Muhafazas 1960-1996, *Journal of Developing Areas* 40-2, 185-210.

[18] Bonneuil N. and C. Dassouki (2006) Women's Education and Diffusion of the Fertility Transition: the Case of Egypt 1960-1996 in 4905 Administrative Subdivisions, *Journal of Population Research* 23(1), 27-39.

1.4.1 The Baby-boom/Baby-bust in the twentieth Century

The history of fertility fluctuations in the twentieth century has been thought in terms of cycles notably since the scenario suggested by economist Richard Easterlin, who conjectured the existence of cycles. Ronald Lee and Kenneth Wachter have suggested models in terms of integro-differential equations, which, in the neighborhood of a certain equilibrium and with certain parameters, produces self-sustained oscillations, which would guarantee a perfect prediction of fertility. However, only a long stagnation has been observed after a single 'cycle', the after-war *baby-boom* and *baby-bust*. Historical records provide no evidence to cycles in population matters. Moreover, the mathematics of time allows us to study temporal signals without a priori periodic scenarios in mind, at least since PoincarÃľ at the beginning of the twentieth century. Phase-space analysis is highlighting: it shows that fertility time series travel between attractors, identifying fertility regimes. The phase portrait of fertility since 1930 and its first-return map reveal the rapid rise of fertility from a low regime to a higher regime, a temporary stagnation, a slow return to the pre-war level. Instead of a linear succession in a formal chronology, phenomena jump from one regime to another one. Where typologies of time series were listed, I showed that a certain unity of temporal forms characterizes the history of European trajectories.[19]

This work opened the road to re-reading the past in terms of dynamical systems. Where Easterlin suggested a mechanistic relationship between economic well-being and reproduction, we know myopic agents in an uncertain environment. It is then necessary, contrary to certain popularized ideas in physics according to which simple equations generate complexity, to highlight simple dynamics in the complexity of the social world. I have supported the phase-space analysis of fertility 1930-1989 by an explanatory scheme

[19] Bonneuil N. (1990) Contextual and structural factors in fertility behavior, *Population, English Version*, 69-92.

based on inertia constraints of economic variables and maintenance or collapse of lifestyles. The trade-off between consumption and parity delineates a line of states beyond which families will be impoverished unless reproduction is reduced. This boundary is attained when one increases her consumption more quickly than her wealth for a given family size. This model allows discontinuous and unpredictable trajectories, which fit observed data.[20]

1.4.2 The Life Cycle

Viability in the economic life cycle I resumed the question of the life cycle in a critical perspective of the economic life cycle according to Friedman, Modigliani, or Carroll, from the 1997 patrimony survey and with the modern viability theory, where I suggested an alternative to inter-temporal utility optimization.[21] Having children, guaranteeing a certain way of life, and retiring with a certain capital leaves room for many trajectories, where couples are torn between prudence for old days and for children, and impatience to consume, under the threat of unemployment or of bad returns of their saving. Agents' heterogeneity is rendered both by the whole state space where each state corresponds to a different situation, and by the set of attainable states, which reflects uncertainty inherent both in decision-making and in external shocks. The delineation of all states from which this program in consumption, reproduction, and saving, against the battling of age, is manageable, identifies in return the timely decisions of when have children, when and how much to consume and save. The discontinuous aspect of consumption when a child is born

[20]Bonneuil (1994) Capital accumulation, inertia of consumption, and norms of reproduction, *Journal of Population Economics* 7, 49-62.

[21]Bonneuil, N. and P. Saint-Pierre (2008) Beyond Optimality: Managing Children, Assets, and Consumption over the Life Cycle, *Journal of Mathematical Economics* 44 (3-4), 227-241; Bonneuil N. (2012) Maximum under continuous-discrete-time dynamic with target and viability constraints, *Optimization* 61(8) 901-913.

is fully taken into account through the continuous-discrete differential inclusions. Empirical insight from the 1997 patrimony survey validates the theory; international comparison shows that lower fertility is associated with smaller sets of timely decisions. Instead of looking at the determinants of fertility as usual, I suggest to delineate the set of states from which a given parity is attainable, then to determine how this set varies with explanatory variables.

Viability-Optimality in the economic life cycle I resumed the problem of the economic life cycle to compute the viable states from which the optimal trajectories with respect to an inter-temporal utility start[22]. households starting from high wealth also start from high consumption; households starting with low wealth must consume relatively little at the beginning. The household can increase both wealth and consumption, with jumps in consumption, associated with drops or slowing down of wealth accumulation. At mid-life, the maximal-viable path consists of consuming regularly more until the end of life span, with the associated decrease of wealth. The maintenance of the trajectory within constraints is of major interest. What is the use indeed of an optimal trajectory which would leave households ruined or starving? This is the motivation for searching for viable-optimal solutions.

Pay-as-you-go Pensions are threatened by the arrival of numerous age classes born during the baby-boom, and followed by less numerous age classes. The numerical unbalance between payers and pensioners is emphasized by the regular lengthening of life expectancy. The usual procedure is to simulate scenarios a priori, in exploring a (necessarily small) set of plausible dates and values on the change of important variables, in deleting those which drive the pay-as-you-go system bankrupt, and with fingers crossed, that at least

[22]ibid: Bonneuil, N. (2011), Maximum...

one attempted scenario works. However, the dimension of the space of possible changes is the number of the state variables: for example five when one considers the duration and the amount of spending, the interest rate, the rate of unemployment, and the ratio of the number of pensioners over the number of payers. This ratio varies in time with mortality, migrations, and the entry into activity of younger age classes. In a two-dimensional space, it is already very long to try all scenarios of changing these variables. The other procedure I developed[23] consists in determining all states from which there exists at least one solution allowing the maintenance for long enough for pensioners and active, while respecting a certain equity between generations, without understating the resistance of social agents to change. The actions necessary to maintain the system within specified constraints are notably the lengthening of paying duration and the increase of payment, for fixed unemployment and interest rates. One result of the algorithm is to indicate which are the viable decisions, when to put them in action, and with which magnitude. The difference with classical optimization procedures is the taking into account of constraints, the opening of the answer to all viable constraints, and the inclusion of constraints in the search for an optimum.

1.4.3 Pension funds

I examined the sustainability of a pension fund, how the long-term consequences of an error in the allowed benefits through bonus points is multiplied by the age structure and the fluctuations on the stock market.[24] I established a relationship between the minimal time to insolvency and the liability/assets ratio.

[23] Aubin J.-P., Bonneuil N., Maurin F. and Saint-Pierre P. (2001) Viability of Pay-As-You-Go Systems, *Journal of Evolutionary Economics* 11, 555-571.

[24] Bonneuil, N. (2013) Early warning to insolvency in the Pension Fund: the French case, *Risks* 1, 1-13.

Generational Equity Similarly, preserving equity between generations consists in regulating the transfers between generations so that no generation spends more than it receives: I resumed this problem in avoiding to optimize an inter-generational utility over an infinite future, but in expressing the bare maintenance of inter-generational equity. I showed the existence of an optimal return rate of human capital: for low values (between 4 and 11 % in the French case), a net present value for each generation increases with the rate of return of human capital and with the viability kernel; but too high a rate of return (beyond 11 %) brings about a shift between pensioners and active and leads trajectories out of the equity constraint.[25]

Vintage models I revisited the vintage model of economic growth from the point of view of viability compared with optimality.[26] The viability kernel shrinks with scrapping time as the technological rate increases, showing that machines must be renewed in line with this rate in order to maintain consumption. The viable and optimal solution in the sense of inter-temporal consumption is obtained on the viability boundary under an auxiliary system.

Thanks to my viability algorithm (2006), I could address the question of the viability and optimality in vintage models, which have the technical difficulty of the presence of a lagged effect.

The Origin of Preferences The origin of preferences was viewed as related to the dominant eigenvalue of a Leslie matrix modelling reproductive strategies. However, in a

[25]Bonneuil N. and R. Boarini (2004) Preserving Transfer Benefit for Present and Future Generations, *Mathematical Population Studies* (G. Feichtinger and V. Veliov editors) 11(3-4), 181-204.

[26]N. Bonneuil, N. (2010) Viability and Optimality in Vintage Models, in R.Boucekkine, N. Hritonenko, Y. Yatsenko, *Optimal Control of Age-structured Populations in Economy, Demography, and the Environment.* New York Taylor and Francis, pp. 108-125.

variable environment, the coexistence of varying preferences no longer requires optimality, but is identified to the mathematical property of viability.[27]

The coexistence kernel of two competitors with varying preferences is computed in the case of scalar and 2×2 Leslie matrices, with either measurable or differentiable preferences. The homologue of indifference curves is the regulation map, which is the correspondence associating the set of viable preferences to a given state of the population.

Among these viable trajectories, some are also optimal in the sense of dominance discounted in time. These viable optimal solutions are obtained as specific trajectories in an auxiliary dynamic system, and the associated maximal values constitute one boundary of the viability kernel of this auxiliary system.

Hence, the perpetuation of varying preferences allows the economic diversity of preferences, as the comparative history of fertility in nineteenth century France and England taken as an example shows.

The Ramsey model I re-visited[28] the Ramsey model of economic growth is revisited from the point of view of viability compared to optimality. A viable state is a state from which there exists at least one trajectory in capital, consumption, and reproduction that remains in the set of constraints of minimal consumption and positive wealth. Viability is first presented with a constraint of minimal consumption, then with an additional criterion of economic sustainability in the sense of the Brundtland commission, which amounts to requiring a non-decreasing social welfare. The comparison of viability kernels with or without sustainability shows how much consumption should be reduced and when. One strong mathematical result is that the viable-optimal solution in the sense of

[27] N. Bonneuil (2010) Diversity of Preferences in an Unpredictable Environment, *Journal of Mathematical Economics* 46, 965-976.

[28] Bonneuil, N, and Boucekkine, R. (accepted) Viable Ramsey Economies.

inter-temporal consumption is obtained on the viability boundary of an auxiliary system. I could compute the optimal paths in a 5-dimensional problem including sustainability criteria in the sense of the Brundtland commission. Varying preference, technological, and demographic parameters randomly over simulated viability kernels with and without the Brundtland criterion help identify the determinants of the non-emptiness of the viability kernel and of its volume: technological progress works against population growth to favor the possibility for a given state of being viable or viable-sustainable.

Economic sustainability as defined by the Brundtland commission adds the restrictive criterion of non-decreasing social welfare. The viability kernel of sustainable solutions shows the necessity of limiting one's consumption, by how much, and when. The viable, optimal, and sustainable solution is computed on the viability boundary under an auxiliary system that combines optimality and constraints.

1.4.4 Single mothers have a leisure premium

With Olivia Ekert-Jaffé, we[29] used the collective representation of household behavior implying a Pareto equilibrium between spouses to show that single mothers are likely to have more leisure than women in couple, a result consistent with a French survey on women's time use, and consistent with common sense (although not with clichés). Moreover, the Pareto-optimal leisure of the man is a decreasing function of the Pareto-optimal leisure of the woman, implying that women the most deprived of leisure time are better off when separated (with respect to leisure) and their former partner are worse off. The consequence for children is ambivalent, depending on the position of the equilibrium of separated spouses. The extension of the model to household production shows that the variation of the time devoted to children between mother in couple and single mother is

[29]Bonneuil, N. and Ekert-é () Single mothers have a leisure premium

also ambivalent; and the various cases are detailed.

1.5 Viability in Dynamical Social Networks

Regulations can also be connection matrices. The success of network analysis in social sciences relies on the statement that certain social events which are poorly explained from socio-economic explanations become clearer when they are linked to networks of relationships. The now classical procedure is to estimate connection matrices. Some authors have suggested to include socio-economic variables, and to make out dominance or substitution relationships in the construction of networks. This amounts to considering social networks as variables in spaces of networks. I have suggested a different view[30] on dynamical networks, showing that they appear less as state variables than as controls in controlled systems. In Sampson's benchmark study of monks, I showed that the state variable is the willingness to stay in the monastery. When a crisis occurs, the network of friendships or antagonisms rules the order of departure of the monks. The social crisis is then a viability crisis of the system. Thanks to viability conditions, I reconstituted the underlying processes leading to the observation of the event. I reconsidered a second famous example showing that the political control of the Medici in 15th century Florence ("everyone knew that the Medici wanted, as bankers, to make money; as families, to increase prestige; as neighborhood patrons, to amass power") resulted from a network of business and matrimonial network. Padgett and Ansell (1993) showed that the Medici drew their power from the central place they occupied in the network. I showed that the network actually used satisfies the viability conditions of the domination of the Medici. These conditions determine all networks which would have made possible the same success:

[30] Bonneuil N. (2000) Viability in Dynamic Social Networks, *Journal of Mathematical Sociology* 24(3), 175-182.

the networks are then strategies in the struggle for power, and the state variable is political power. The challenge is then to use the *viable* networks, thanks to which the temporal trajectory will bring the political control to the Medici: these networks satisfy the viability conditions. I also showed that the centrality of the Medici implies the viability of the network in the framework of political domination.

At each time, a whole set of networks is possible for the Medici, not only the one retained by chronicle: the Medici could have established new links, cease others, a whole set of change was possible, but only those viable were beneficial to the Medici. Their skill was in their ability to establish and select the right links.

1.6 Spatial Diffusion under Control (Mutational Equations)

In social sciences, diffusion has been used to describe the spread of a disease, acquisition of a skill, or social changes resulting from innovations in customs, beliefs, tools, techniques, adopted by one people from another. In natural or social sciences however, processes may draw shapes which are not necessarily regular sets (as they can be in diffusion processes). For example, schooling does not "spread" in the sense of a disease, or in the sense of a custom from one ear to another. The process requires investment in money, in organization, in humans; it is tuned by the demography of pupils. The quality of teaching or total number of children per class also stems from decision-making and resource constraints. Moreover, the differentiation of schooling between girls and boys covers political and societal will; the competition between confessional and secular systems also obeys to the law.

I developed on the concept of derivative of a shape function to study sets and set-valued maps (or correspondences). The concept of graphic derivatives of set-valued maps, and the *mutation* of a map (Gore, 1997) allowed me to define the velocity of a set and kinds of differential equations governing the variation of sets, in the general framework of metric spaces.[31]

1.7 Epidemiology

In epidemiology, I introduced the risk by age in the analysis of genetic components of susceptibility to diabetes.[32] So far, a constant probability with respect to age was associated to combinations of genes, although we know that certain diseases such as diabetes or breast cancer vary with age. This work allowed the evaluation of the effect of genes from the HLA group on diabetes. A software allying segregation analysis and survival analysis was written and made available (in collaboration with Antoine ClergÃľ).

I suggested [33] the construction of ageing laws at the DNA level, from the data of 12000 genes taken post-mortem in the neo-cortex of each of 30 individuals. During life, certain active genes since birth turn down, while others turn up to supplement failing genes. The difficulty of knowing which gene has transited from up to down or reverse adds up to the difficulty of collecting data (to which I took no part), and to the difficulty of computing up or down transition risks when individuals are deceased (we do not know when the event took place), the difficulty to take into account the multi-level dependence between genes, coming from the fact that they belong to a same individual, and the difficulty to pass from the risk of each gene to the rate of senescence of the whole set of genes for each individual.

In epidemiology, in relationship with the themes of the life cycle, populations and environment, I took part in the study of the statistical foundations of the incubation time

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[31] Bonneuil, N. (2012) Morphological Transition of Schooling in 19th Century France. J. Math. Sociology

[32] Bonneuil N., A. Clerget, and F. Clerget-Darpoux (1997) Variable Age of Onset in Insulin-Dependent Diabetes Mellitus by the Marker-Association-Segregation-χ^2 Method, *American Journal of Human Genetics* July 61, 223-227.

[33] Bonneuil N. (2007) Ageing laws for the human frontal cortex, *Annals of Human Biology* 34-4, 484-492.

of AIDS,[34] to the search of determinants of measles in eastern Senegal,[35] I highlighted the transfer of measles from 0-1 year old boys to girls in Bamako,[36] and showed how dispersion and isolation allowed certain Amerindian tribes to survive to epidemics in Orinoco after the contact with Europeans.[37]

1.8 Controlling Population Biological Systems

1.8.1 Population Genetics

Population Paths of the Distant Past from Genetic Data Insight into the time of populations in the distant past is now made possible by the exploitation of molecular genetic data. The demographic fluctuations of the distant past are involved in the resulting heterogeneity of the genome, in particular that of mitochondrial DNA with a mutation rate higher than nuclear DNA. Does the distribution of pairs of nucleotides conceal the track of a "bottleneck" which would have occurred thousands years ago, as it was claimed? Monty Slatkin showed that the samples are all compatible with the assumption of a constant population, as with the assumption of an exponentially growing population.

I suggested to find [38] the whole set of temporal paths capable of producing the observed

[34]Brouard N. and N. Bonneuil (1992) How reporting delay, duration of follow-up and number of cases affect the estimates of the incubation time of transfusion-associated AIDS, *Math. Pop. Studies* 3(3), 189-198

[35]Pison G. et N. Bonneuil, (1988) The Impact of Crowding on Measles Mortality. Evidence from Bandafassi Data (Senegal), *Review of Infectious Diseases*, vol. 10/2, 468-470

[36]Bonneuil N. et P. Fargues (1989) Predicting the vagaries of mortality: causes of deaths in Bamako 1974-1985, *Population Bulletin of the United Nations* 28, 58-94

[37]Mansutti Rodriguez A. et N. Bonneuil (1996) Dispersiòn y asentamiento interfluvial: dos razones de sobrevivencia étnica en el Orinoco Medio del post-contacto, *Antropologica* 84, 43-72

[38]Bonneuil N., (1998) Population paths implied by the mean number of pairwise nucleotide differences among mitochondrial DNA sequences, *Annals of Human Genetics* Jan 62, 61-73.

genetic heterogeneity (polymorphism). This is a problem of viability: I looked for the largest set of states from which there exists a solution leading to the observed result, which here is the confidence interval of the total number of genetic differences within a sample of individuals (nucleotide differences in a pair of genes drawn at random). The solution I am mentioning is the one of the "coalescent" process: knowing the mitochondrial DNA of individuals, one can compute the probability that two of them have a common ancestor a certain number of generations ago. This probability depends on the history of the population size. Moreover, the genetic differences of two individuals come from the sole mutations occurred since the common ancestor. Hence the relationship between genetic differences and demographic fluctuations in the past, which however remains hard to handle when the population varies in time.

Instead of validating a plausible or appealing scenario such as the famous demographic "bottleneck," I computed the domain of possible histories reflected by the data.

1.8.2 Demography Inferred from Cemeteries ('Paleodemography')

Another case study comes from paleo-demography: it was attempted to deduce fertility and mortality from the age distribution (whose very determination is matter of controversy) of skeletons found in old cemeteries. The simplest scenarios, such as the stationary or the stable population models fit poorly to data. Demographic paths as simple as these are unlikely to have existed, for example if we keep in mind the very irregular fluctuations of reconstructed fertility and mortality in 17th century France. On the contrary, my position was to notice that the age distribution of skeletons gives a mere hint at the set of demographic trajectories, those passing through the demographic states of the capture domain of this distribution, which now appears as a target in an appropriate state space, under Lotka-McKendrick dynamic. Among these demographic paths producing the observed distribution of deaths by age, one of them is the most parsimonious in terms of fertility and mortality fluctuations, and of deviation with respect to the stable age structure. Finding this solution requires to leave the stable framework, at the price of a dramatic augmentation of the number of degrees of freedom (from 2 in the stable case to 25 in the empirical case of Belleville).

This is what I did with modern techniques of stochastic optimization.[39] I showed that the mean life expectancy and the mean fertility are reconstructed correctly, and I could estimate these measures in the case studies of St. Thomas Anglican Church (1821-74) (Belleville, Ontario) and Dallas Freedman's Cemeteries (1869-1907) (Dallas, Texas).

Set-valued analysis, which I pioneered in demo-economy and population genetics, allows me to reorganize the concept of trajectory, to contribute in rehabilitating the connection between mathematics, historical time, and narrative, to extend the paradigm of historical discourse out of the categories of "probability-improbability" (White, 1973) to those of "viability-non viability" in a context of uncertainty.

1.9 Theorems and Algorithms

1.9.1 Existence and Uniqueness of Solutions to Lotka-Volterra integro-differential System

Webb (1981) showed the existence and the uniqueness of the solutions of the system constituted of the McKendrick differential equation and Lotka integral equation, at the heart of the mathematics of populations,[40] in the framework of integrable solutions in

[39] Bonneuil N. (2005) Fitting to a distribution of deaths by age with application to paleodemography, *Current Anthropology* 46, 29-45.

[40] On which I wrote a textbook: Bonneuil N. (1997) *Introduction Ãă la modélisation démographique*, Paris : Armand Colin, Collection U, 128 pages.

the L_1 sense. Suggesting a set-valued perspective of social time, I ought to revisit the foundations of demography. With Jean-Pierre Aubin and Franck Maurin, I extended the theorem of existence and uniqueness of solutions to the case of functions with closed graph.[41] Notably, the solutions appear now as the attainable sets issued from the initial pyramid and births. The set of these attainable sets is *the invariance envelope* (which is also the capture domain in reverse time of the initial pyramid and births), whose existence and uniqueness result from purely topological properties. The respect of constraints by the system throughout time comes from the very construction of the invariance envelope. The mortality, migration, and fertility forces are very general.

1.9.2 The Viability Algorithm in large State Dimension

I addressed the computation of viable states and of the viability kernel in large state dimension, based on stochastic optimization. The idea is to minimize the distance to the set of constraints of solutions starting from a given state, and to assess the viability status of this state whether or not the minimization of the distance leads to at least one trajectory remaining in the set of constraints. The search for viable states is also achieved by the minimization of a distance to the set of constraints, so that the procedure relies on a double stochastic optimization: one where the initial state under examination is fixed, so as to decide whether it is viable or not, and one where this initial state is varied.

[41] Aubin J.-P., Bonneuil N. and Maurin F. (2000) Non-linear Structured Population Dynamics with Co-Variates, *Mathematical Population Studies* 9(1), 1-31.

1.9.3 The Maximum under Viability Constraints in discrete-continuous (hybrid) time

In the problem

$$\begin{aligned} &\int_0^T L(x(t),u(t))\,dt \\ &x'(t) \in F(x(t)) \quad \text{a.e. in } [0,T],\ x(0)=x \end{aligned} \tag{1}$$

I first[42] contested the unproven claim by Aubin extrapolating Cannarsa and Frankowska's result on the viable minimum that the maximum is achieved on the boundary in the direction of low y of the "absorption-viability kernel" of K in $K \times I\!R^+$ under the extended dynamic $(x'(t), y'(t)) \in (F(x(t)), -L(x(t),u(t)))$. My counter-example has an empty absorption-viability kernel; yet, the viable maximum exists and is easy to compute.

Second, I showed that the viable maximum of $\int_0^T L(x(t),u(t))\,dt$ of a continuous function $L \in \mathcal{L}^1(I\!R^{2m+1}, I\!R^+)$ under a dynamic $x'(t) \in F(x(t))$ under constraint $x(t) \in K$ where K is closed is obtained on the boundary of the capture-viability kernel in direction of high y of the target $K \times \{0\}$ viable in $K \times I\!R^+$ under the extended dynamic $(x'(t), y'(t)) \in (F(x(t)), -L(x(t),u(t)))$. The result holds true with discrete-continuous-time measurable controls.

1.10 The Time of Populations and the Viability Principle

I commented upon this perspective of time,[43] in situating it with respect to chaos theory imported from physics and with respect to the narrative mode claimed by micro-history. Instead of viewing history as unfolding along a trajectory, I pleaded for set-valued anal-

[42] Bonneuil N. (2011) Maximum under continuous-discrete-time dynamic with target and viability constraints, *Optimization*.

[43] Bonneuil N. (2001) History, Differential inclusions, and Narrative, *History and Theory* Theme issue 40 'Agency after Postmodernism', Wesleyan University, 101-115.

ysis and differential inclusions in social sciences, as being more adequate to human time, because the direction the system can take at each time is enlarged to the set of all admissible directions. These directions do not all lead to a desirable future, and the challenge is often to go to an objective (survival, conquer or keep power, avoid poverty, succeed in a fishing campaign, maintain the diversity,...) in spite of uncontrolled perturbations. I also discussed the contribution of probability theory and dynamical games to History.[44]. I suggested the themes of continuity and connectivity as a red thread in the foundations of narrative in history, and examine various conceptions of dynamics in history telling.[45]

Data of the past conceal an additional difficulty. As each time contains a myriad various trajectories, remains no longer reflect a single past. Historical data can be insufficient to reconstitute a single history, and although this history has actually existed and has been unique, we ought to acknowledge our ignorance and associate to it not the most pleasant or the easier-to-imagine scenario to past data, but the *capture domain*, the set of all states from which there exists at least one trajectory producing the observed data. This is what I developed in population genetics and in paleo-demography.[46]

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[44]Bonneuil N. (2004) Repertoires, Frequentism, and Predictability, *History and Theory* 43(1), 117-123; Bonneuil N. (2005) History and Dynamics: marriage or *mésalliance*?, *History and Theory* 44(2), 265-270; Bonneuil N. (2009) Do historians make the best futurists?, *History and Theory* 48 (Feb), 98-104; Bonneuil N. (2013) Viabilité, probabilités, induction, *Tracés*, 24, 71-84.

[45]Bonneuil N. (2010) The mathematics of time in history, *History and Theory* 49, 27-45.

[46]Bonneuil, N. (2008) The mathematics of maintenance and acquisition, in J. Chen and C. Guo (ed), *Ecosystem Ecology Research Trends*, Nova Science Publishers, 153-175.

1.11 Mathematics for Social and Bio-Sciences?

I then contributed to introduce modern mathematics of time into population dynamics. The stiff framework of stable populations or Markov processes which prevail in demography tells us about kinematics, but not on the time of populations. Simulations, although very popular, are ineffectual to let us know about state space, even in two dimensions. Many models rely on conjectures, where specifying interactions tell a specific story, but seldom result from phenomenological considerations. The survival or the failure of a system constitute interesting principles in social and natural sciences. The mathematics of viability allows us to translate these principles faithfully, and to confront them to constraints so as to infer what these interactions should be in order that the system perpetuates itself or should have been in order to take the shape it has today. Rather than prediction, these set-valued maps or correspondences give the "map of the future": how should actors react in such or such situation if the system is to perpetuate itself. I then suggest a shift in our traditional concepts of stable population, simulation, predictions, probabilities, equilibria, which, besides, I contributed to make alive in demography, toward a conceptual and mathematical framework capable of letting us think on and handle uncertainty, human agency, and transient dynamics, putting history and mathematics of time at the heart of the understanding of demographic transformations.