Differential heterogenesis and the emergence of semiotic function

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This study reconsiders the Deleuzian concept of differential heterogenesis from the point of view of its mathematical basis, that are just sketched in "Difference and Repetition". Differential heterogenesis is at the center of the idea of "becoming" in terms of actualisation of a prendividual field of differential singularities. The passage from the virtual to the actual is formalized as the integration of a differential problem to build a variety of forms.

Unlike usual differential calculus common in mathematical-physics, heterogenesis is based on the assemblage of differential constraints that are different from point to point. The construction of differential assemblages will be introduced in the present study from the mathematical point of view, outlining the heterogeneity of the differential constraints and of the associated phase spaces, that are continously changing in space and time. Our purpose is to free up the dynamic becoming from any form of unitary and totalizing symmetry and to develop forms, action, thought by means of proliferation, juxtaposition, and disjunction devices.

After stating the concept of differential heterogenesis with the language of contemporary mathematics, we will face the problem of the emergence of the semiotic function, recalling the limitation of classical approaches (Hjelmslev, Saussure, Husserl) and proposing a possible genesis of it from the heterogenetic flow previously defined. We consider the conditions under which this process can be polarized to constitute different planes of Content (C) and Expression (E), each one equipped with its own formed substances. A possible (but not unique) process of polarization is constructed by means of spectral analysis, that is introduced to individuate E/C planes and their evolution. The heterogenetic flow, solution of differential assemblages, gives rise to forms that are projected onto the planes, offering a first referring system for the flow, that constitutes a first degree of sensibility.

This construction allows the emergence of the semiotic function from the dynamic evolution of the heterogenetic flow, without the need of any stabilization, in the opposite to structural morphodynamics.

1 Introduction

Semiosis is a generative and emergent process as clearly stated by Paolo Fabbri in [19] where the author outlines that "differently from logic, the point is not to construct preliminary logical systems and then to see how they work in language, but to assemble some very simple units and then observe emergent properties." (Ibid., our trans.) The aim is to "see how, at a certain point, some emergent properties organize themselves and acquire meaning, that is exactly the opposite of old semiotics and old logics." (Ibid., our trans.).

From the point of view of structural semiotics, signification processes presuppose systems of units whose identity is differential and positional. Dynamical structuralism [48, 49, 34] has deeply studied systems of oppositions engendered by qualitative discontinuities in morphodynamics, proposing to model several semiotic systems with catastrophe theory (see [36] for a review). But the ontogenetical issue has to be faced now on new basis taking into account the globality of the morphogenetical process including the emergence of the semiotic function and of the semiotic spaces where systems of oppositions will be installed. Far before stabilization of dynamics in the basins of attraction there is a progressive individuation of the space itself where the landscape of potentials lives. Far before that categorisation is performed by sign [38], a multitude of symptomatic elements contribute to construct a protosemiotic space already equipped by its semiotic function without the presence of any stabilization.

This perspective has been recently reconsidered in [43] from the point of view of the concept of *individuation* as introduced in the work of Gilbert Simondon [46]. With the idea of individuation G.Simondon interprets the becoming of forms as a continuous passage from a pre-individual intensive plane to an emergent plane of extensive forms. In his thesis "L'Individuation à la lumière des notions de forme et d'information'" Simondon considers this process at different scales and levels of complexity: physical, biological, psychological and transindividual.

Deleuze clarifies in “Difference and Repetition” [12] that the individuation process has a differential origin and that the becoming of forms is defined as the solution of a differential problem. Following Deleuze, the differential becoming is a passage from the virtual to the actual, where the virtual is a distribution of differential operators. This distribution is heterogeneous, since the differential operators are all different one to the other and for this reason they are called "singular". This singular distribution is intensive and cannot be perceived, since it does not belong to the phenomenal plane. Just the integration of the differential constraints give rise to forms, perceptions and extensive morphologies in the endless becoming of the differential heterogenesis.

In this study we propose a mathematical formulation of heterogenetic becoming and we analyse how progressive polarisation of the heterogenetic flow allows the emergence of the expression/content spaces.

Rather than making a philological analysis of the deleuzian trascendental empiricism that we leave to specialists (see for example Anne Sauvagnardes in [12]), we prefer to let be inspired by its guiding ideas and to build upon the thinking of a possible mathematics of heterogeneous becoming. Some recent contributions on the role of mathematics in the philosophy of Gilles Deleuze have been as well as of great inspiration for the development of the present study, such as [17, 28, 30]

In the first part of the paper we will outline that, differently from differential processes in mathematical physics, where differential operators are invariant in a certain phase space and they are given as invariant laws, in heterogenesis the differential constraints are singular and are composed to build always different assemblages ("agencement"). The operation of composition of assemblages is similar to an action of editing of differentials, that are modified, added, eliminated and in general recombined in new configurations. This action corresponds to a true plastic assembly of a multiplicity of differentials on the virtual plane. Assemblages define time by time their own spaces. In this case the morphogenetical space is not given a priori as in mathematical physics, but it's a consequence of assemblage of singularities. Together with a morphogenesis *in* the space, we have also a morphogenesis *of* the space, since assemblages are continuously evolving.

In the second part of the study we will introduce the emergence of the semiotic function as progressive polarization of the heterogenetic flow leading to the separation of E/C planes, each one containing its own formed substances. While the dynamics of separation of the two planes is largerly unknown and typical to each semiotic context, we will speculate on the possibility of a spectral differentiation of the planes, where the eigenvectors of the assemblage will indicate the independent directions of subspaces E/C. This construction allow the emergence of the semiotic function from the dynamic evolution of the heterogenetic flow without the need of any stabilitation, in the opposite of the classical case of structural morphodynamics.

2 Elements of Heterogenesis

Gilles Deleuze in Difference and Repetition proposes a concept of becoming that is largely based on the Simondonian idea of individuation. What it means individuation? It means passage from a preindividual field to an individuated one. Precisely Deleuze specifies this passage as from a virtual plane to an actual one, or also in terms of a transformation from the virtual plane to its actualization.

Deleuze, differently from Simondon, characterizes this passage in a specific mathematical way. He reconsiders the concepts of differential of Leibniz and defines the virtual as a multiplicity of differential operators. This distribution is intensive and not perceivable. The perceived forms, as well as the mental forms of thinking and more in general any morphogenetical process is nothing but the solution of a problem posed by the multeplicity of differential constraints that constitute the virtual. In other words the origin of any morphogenesis is differential.

Even if differential calculus is just a mathematical tool, the differential becoming is considered as a general dialectic that overcome mathematics: « il trouve son sens dans la révélation d'une dialectique qui dépasse la mathématique »[38]. There are problems that are mathematical, physical, biological, sociological, semiotic who find their solution in the different disciplines by actualizing differentials in a proper manner. In any case the solutions emerge always by integration of « un système de liaisons entre éléments différentiels, un système de rapports différentiels entre éléments génétiques. Si l'Idée est la différentielle de la pensée, il y a un calcul différentiel correspondant à chaque Idée, alphabet de ce que signifie penser »[42].

The classical example of such a dfferential problem is the Riemaniann manifold, that is constructed by the integration of a family of tangent planes, each one carrying its proper differential constraint wich coincides in the intersection. In the Riemannian problem the differential constraints are homogeneous, or more precisely equiregular, but nothing prevents to leave more freedom to differential operators and consider heterogenous constraints. This case is just skeched in Difference and repetition and it is developed together with Felix Guattari in Thousand Plateaus. The concept of heterogeneous assemblage (agencement) of planes is introduced in Thousand Plateaus [11] at a philosophical level more than at the mathematcal one, but it is clearly the extension of the idea of differential becoming to a heterogeneous setting.

We freely interpret this heterogeneity at least from two different perpectives. We find a first level of heterogeneity in the constitutive difference of differential contraints, that can induce a variety of dynamical behaviours changing point to point. Furthemore a second level of heterogeneity is present since each differential constraint has its own structure of tangent planes constituing the phase space, that are the “plateaux” where fluxes are allowed to flow on. The continuosly changing geometry of directions of the flows is then a further elements of heterogeneity.

This heterogeneous differential problem is posed in terms of a composition of the differential constraints to form assemblages. Heterogeneous assemblages are not built on the base of a logic compatibility or complaiance, but by the possibility of differential constraint to create new spaces and new dynamics not given a priori, in such a way that phase spaces as well as dynamics are invented by the intrinsic construction of the singular composition. How this heterogenous composition is possible is one of the mathematical problem we would like to face in this study. How the conjunction of singular differentials is able to give rise to an agencement is a difficult mathematical problem that we will consider in the following starting from the work of Rothschild and Stein [39].

Just to envisage what a similar approach can carry on, let’s consider the organization of brain dynamics. The brain is made up of neural populations with heterogeneous dynamics that are mathematically described by heterogeneous operators. At the same time, populations act on a set of neurochemicals such as neurotransmitters, messengers and neuromodulators giving rise to a heterogeneity of formed substances. Again neural connectivity that defines the structure of tangent planes of dynamics is different population by population. These populations are concatenated together in the form of ’agencement’, then they must therefore be considered as a material implementation of heterogenesis. Finally neural connectivity is plastically modified by learning processes implementing a true plasticity of the virtual, that is a continous reorganization of the differential rules that underly dynamics.

Brain heterogenesis therefore constitutes the material differential layer of every phenomenology of perception and imagination, whose forms are deployed as a solution to the differential problem (notice that Deleuze and Guattari face this topic in their last work “What is philosophy?” [13], to which we refere the interested reader).

We are quite far from the usual differential calculus of mathematical physics where the distributions of operators is spatially and temporally homogeneous, while in heterogenesis there is a spatially and temporally varying definition of differential constraints. Mathematical physics is a form of symmetrization of heterogenesis in the sense that any heterogenous set is reduced to a unique operator that holds in every spatio-temporal point. Heterogenesis can be regarded as a *Hyperphysics* that takes place as a variety of dynamics flowing on a multiplicity of tangent planes that change point to point.

This character of “homogeneisation” of mathematical physics is at the base of its fundamental a-priori, who presupposes that spaces are given a priori with respect to differential constraints. This a-priori is completely reversed in the composition of heterogenetic assemblages, where operators are primary and define dimensions and qualities of the space: a new differential singularity that is composed with a assemblage redefines completely the spaces of the entire assemblage.

In mathematical physics operatorial homogeneity and the fixity of the differential constraints determine the universality of laws and the nomological character of differential models. Heterogenetic composition is pole apart from universal laws and lays the conditions for an immanent morphogenesis that is created time by time by the assembly of singular concatenatations.

Notice that if the assemblage of operators is considered in turn as a new differential operator, heterogenesis can be viewed as a morphogenesis of the assemblage operator. The heterogenetical becoming is then to be considered as a concurrent morphogenesis of operators, of its spaces and of forms in spaces, that is unprecedented in physical and structural dynamics. Composition of singular assemblage as to be tought as a true invention, the creation of new dynamics instant by instant. The inventive character of the assemblage is due to the fact that the space created by the assemblage is much more than the union of identitary spaces of single operators. As we will clarify in the mathematical presentation that follows this is due to the facrt that second order differences (difference of differences) increases the dimension of the tangent space and open to new planes, inconceavable before.

As Michel Foucault explains in the introduction to Difference and Repetition [12], rather than search the common under the difference, it is to think in a differential way the difference. And precisely because of these differences of differences (that occur through the commutators) new spaces arise with all their possible dynamics.

What is the meaning to reconsider today heterogenesis from the mathematical point of view? The first motivation relies on the fact that the very origin of deleuzian heterogenesis has an operational nature, since Deleuze takes as a model differential calculus of Leibnitz and more in general the operational disposition of baroque culture. Differential calculus is at the base of the idea of becoming in Difference and Repetition. Becoming assumes from the beginniing a problematic dimension, in the strict mathematical sense to pose and solve a problem.

Deleuze explicitly explains the role of mathematics in its constructivist empiricism:

“ ... how can something be given to a subject, and how can the subject give something to itself? Here, the critical requirement is that of a constructivist logic which finds its model in mathematics. The critique is empirical when, having situated ourselves in a purely immanent point of view, which makes possible a description whose rule is found in determinable hypotheses and whose model is found in physics, we ask: how is the subject constituted in the given? The construction of the given makes room for the constitution of the subject. The given is no longer given to a subject; rather, the subject constitutes itself in the given." (Empiricism and subjectivity, [15] p. 87).

Becoming is view as the creative principle arising from the position of a problem in terms of a constellation of differential operators heterogenous among themselfs. This phase of plastic composition of differentials put in place the problematic and intensive dimension of becoming, that can be regarded as a form of plasticity of the virtual. Mathematics can then be used as a language to evoke the becoming of a complex materiality endowed by its substantial consistency as a vital, singular, semiogenetic flow (about this idea of vital materialism see also Rosi Braidotti [4]).

Beside this intrinsic motivation, there is also a historical contingent factor that push us to elaborate on heterogenesis. As in the epistemic view of Albert Lautman, mathematics is considered here as a language that is always relative to specific and situated problematic circumstances, where an important part of mathematical invention consists in the formulation of problems. History of mathematics is considered here a history of problems, more than an automatic progress independent from the cultural and historical context, as in the axiomatic perspective. The work of mathematicians is then the one to envision the entire problematic dimension in an original way. We are then interested in the question of heterogenesis more to problematize than to offer solutions. Particularly we are interested to problematize the question of contemporary models in life science and human science. Models in life sciences and human science, from the cognitive to the social point of view, from the aesthetic to the semiotic aspect, come from a culture of physical science in which an invariant and homogeneous distribution of operators is considered. This nomological use of operators is at the base of contemporary modelling culture: the Navier-Stokes equation for viscous fluids is the same in all points of space and time. Analogously the equation of morphogenesis of Alan Turing [50], deeply studied also by René Thom, presents spatial and temporal symmetries. In life science a deep problematization of invariances and symmetries has been proposed by Giuseppe Longo and we refer the reader to his work [1, 18]. In the same way, models of mathematical and computational economy are based on the interaction of individuals endowed by the same space of rationality. These approaches are founded more or less explicitly on the paradigm of methodological individualism [29, 35], where every process of individuation is reduced to a functional interaction between homogeneous units already individuated.

If homogeneous constraints well describe a form of swarm intelligence or crowd behaviour, it reduces dynamics to automatisms, by excluding any form of imaginative and creative aspect. With this study we aim to problematize the procedure of homogeneization that is dominant in life and social science and to outline the dynamical heterogeneity of life and its affective, semiotic, social, historical aspects. The purpose is to free up the dynamic becoming from any form of unitary and totalizing symmetry and to develop forms, action, thought by means of proliferation, juxtaposition, and disjunction dispositives.

3 Heterogenesis as a mathematical differential problem

In usual mathematical setting [21, 20] and particularly in mathematical physics a differential problem is assigned by defining an operator with uniform properties on a domain Ω and the solution is computed by integration given suitable initial and boundary conditions. A different point of view has been recently considered introducing operators heterogeneous in space and time. A class of operators which can have different behavior from one point to an other have first been studied by Hörnander in [25], and then by Rothshild and Stein [39]. Rothshild and Stein defined a multiplicity of operators $(A_{p_i})_{i=1,2,\cdots}$ assigning an operator A_{p_i}, around each point p_i, and defining a new operator A interpolating the given operators. In their work all the operators $(A_{p_i})_{i=1,2,\cdots}$ have the same form, but are defined on differential structure which can change from a point to the other. A later litterature has originated by their work: (see for example [8, 9, 10]) and the review paper [5].

Here we remove the assumption that all the operators have the same form and weaken some requirements of the differential structure. In this way we will define the assemblage operator A ('angencement') starting from a molteplicity of differential constraints that defines the heterogeneity of the virtual.

The actualization of the assemblage operator A will be a flow u whose domain will be a posteriori defined, and depends on the differential properties of the assemblage A. In this way the space of the solution is induced by the operator itself.

3.1 Properties of each operator of the multiplicity

Let us describe a general operator A_{p_0} in the considered multiplicity of heterogeneous operators. In general it depends on directional derivatives of any order of a function u defined in a neighborhood B_{p_0} of the point p_0 (see fig. 1). First order directional derivatives will be denoted $\nabla_{p_0} = (X_{1,p_0} X_{2,p_0}, \cdots)$, higher order derivatives, obtained applying k times first order ones, will be denoted $\nabla^k_{p_0}$, so that the expression of the operator becomes:

$$A_{p_0}(u)(p) = A_{p_0}(p, u(p), \nabla_{p_0} u(p), \nabla^2_{p_0} u(p), \cdots, \nabla^k_{p_0} u(p)).$$

Since the higher order derivatives are obtained applying in sequence first order derivatives, the direction of propagation of the flow mainly depends on $\nabla_{p_0} u$. In the Riemannian setting, all the directions of propagation are allowed, with different velocities. Here we consider more general differential constraint, generalizing the approach of Bony [3] and Hörmander [25]. Hence we assume that the set of allowed directions of propagation $(X_{1,p_0}, X_{2,p_0}, \cdots)$ can change from a point to an other, even within a neighborhood of a fixed point p_0. The admissible tangent space T_p is generated by the operators $(X_{1,p}, X_{2,p}, \cdots)$ at any point p of the neighborhood, hence will have dimension different from a point to the other. The space is called non equiregular.

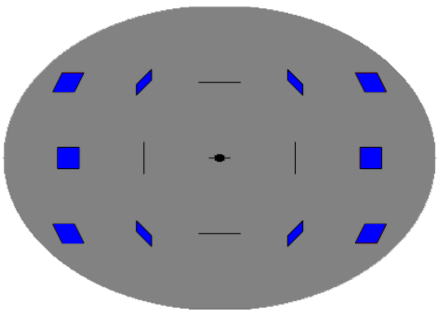

Figure 1: Visualization of the tangent planes T_p induced by an operator A_{p_0} at all points p in a neighborhood of the point p_0. They differ from one point to an other and they can be a line or a plane of any dimension.

3.1.1 The lifting process

In classical mechanics, a lifting process defines the phase space, associating to each point, its tangent space. For example to every point of the three dimensional space it is possible to associate its velocity and the resulting structure has dimension 6 at every point.

Here we consider a more general setting. The admissible tangent spaces T_p at every point are different, they can reduce to a point or to a line, and are no more sufficient to completely describe the direction of motion (see fig.1).

Indeed the flow will propagate along the directions of the admissible vector fields and new vector fields, called commutators, which can be formally expressed as combinations of differences of tangent fields, i.e. second order derivatives (see fig.2). These vector fields denote possible directions of propagation not necessarily contained in the generators. The algebra L_{p_0} contains all the admissible fields X_{i,p_0} and their commutators, and will allow a complete description of the direction of propagation. Indeed flux associated to any operator expressed in terms of these vector fields will propagate not just along the directions of admissible vector fields X_{i,p_0}, but also along the direction of commutators, with different speed.

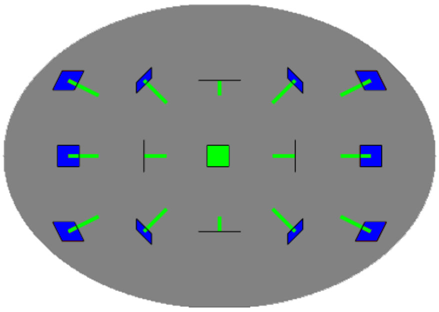

Figure 2: Visualization of the Lie algebra in all points: it contains all the admissible vector fields and their commutators. The admissible tangent plane is represented in blue and the commutators in green

We will now introduce a lifting space, whose tangent space has the dimension of the algebra at every point. In order to achieve this differential property, a delicate construction is applied: it is not sufficient to perform a Cartesian product, but some identifications and quotients on the spaces are needed. Each point p of B_{p_0} is lifted to a point (p, q),and the domain B_{p_0} is lifted to a higher dimensional domain (B_{p_0}, F_{p_0}), where tangent space and its algebra has the same dimension. The gradient in the lifted space will be denoted $\nabla_{p_0,0}$ and the operators A_{p_0} at every point will be lifted to operators $\tilde{A}_{p_0,0}$. In fig.3 is visualized the lifted space and the new family of tangent spaces $T_{p,q}$ at every point, in terms of a multiplicity of planes defining the possible directions of flow of the solution $u(p, q)$.

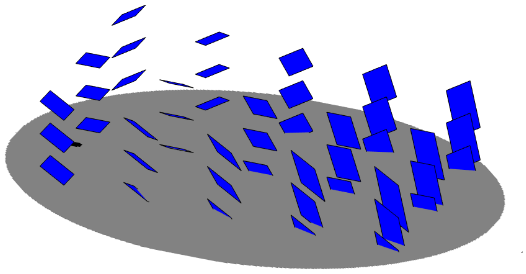

Figure 3: The higher dimensional lifted space and its tangent spaces $T_{p,q}$ at every point.

More general lifting is performed adding variables present in the operators, which can not be differentiated. In this case the lifting can be obtained through differences, leading to more general lifted spaces, always denote (B_{p_0}, F_{p_0}).

This process is different from the one proposed in Hoffman [24], Petitot [33], Citti-Sarti [6, 7], Duits [16] to build neurogeometries. It is more general. In facts in the present case the algebra induced by operators is not necessarily related to a group structure and can be chosen with more freedom. In other words the differential constraint is primary and not deduced from more sophisticated structures.

We can as well assume that gradients ∇_{p_0} which describe the direction of propagation are not a priori fixed. They depend on the dynamic evolution of the solution u. This implies that the vector fields are generators of the solution, and at the same time depend on the solution. The structure of tangent planes will be different if the solution has different values. Equations of this type can present shocks and crack formation: a crack is a sudden episode, non reproducible in the same way.

3.2 The assemblage operator

In the previous section we have defined a multiplicity of operators A_{p_i}. We will see how to construct an assemblage of this multiplicity performed by the operator A, such that $Au(p_i) = A_{p_i}u(p_i)$ for every point p_i of the set P.

As we previously mentioned, the differential operators and the lifted structure are well defined only in a small neighborhood of each point p_0. They are not globally defined.

If we assign two points, p_0 e p_1, they can be connected only if the associated neighborhoods B_{p_0} and B_{p_1} have a nonvanishing intersection (see fig. 4). The two bases (B_{p_0}), (B_{p_1}) will be lifted with new fibers $(B_{p_i}, F_{p_i})_{i=0,1}$ and the associated lifted operators will be $\tilde{A}_{p_0}, \tilde{A}_{p_1}$ (fig. 4 top and center). However in the intersection $B_{p_0} \cap B_{p_1}$ we can collect all the directional derivatives of the two different gradients in a new gradient $\nabla_{p_0,p_1} = (\nabla_{p_0}, \nabla_{p_1})$ (fig. 4 bottom). As previously described we have to consider all their commutators, and to apply a lifting procedure to describe all possible direction of motion. The lifting $\tilde{\nabla}_{p_0,p_1}$ contains commutators which did not exist in each of the lifted operators separately.

In order to regularly evolve from one operator to the other we will associate a partition of unit to the two sets B_{p_0}, B_{p_1}, In other words we define two regular, nonnegative functions ϕ_0, ϕ_1. Each of these functions ϕ_i $i = 1, 2$ has value 1 respectively in its assocated set (B_{p_i}, F_{p_i}) minus an ϵ-neighborhood of the boundary, and 0 outside (B_{p_i}, F_{p_i}). In addition we can require that $\phi_0 + \phi_1$ is 1. This means that inside the neighborhood (B_{p_i}, F_{p_i}) and far from the boundary only the function ϕ_{p_i} is non zero and it identically takes value 1. In the intersection of $(B_{p_0}, F_{p_0}) \cap (B_{p_1}, F_{p_1})$ both functions ϕ_{p_0} and ϕ_{p_1} are nonvanishing but their sum is 1.

Then we can define the assemblage operator

$$\&\tilde{A}_{p_0,p_1} := \phi_{p_0}\tilde{A}_{p_0} + \phi_{p_1}\tilde{A}_{p_1}.$$

This satisfies

$$\&\tilde{A}_{p_0,p_1}u(p_0) = \tilde{A}_{p_0}u(p_0), \quad \&\tilde{A}_{p_0,p_1}u(p_1) = \tilde{A}_{p_1}u(p_1)$$

In the intersection of two domains it smoothly changes from one operator to the other.

The operator can at this point be reprojected to an operator $\&A_{p_0,p_1,\cdots,p_k}$ on the substrate space.

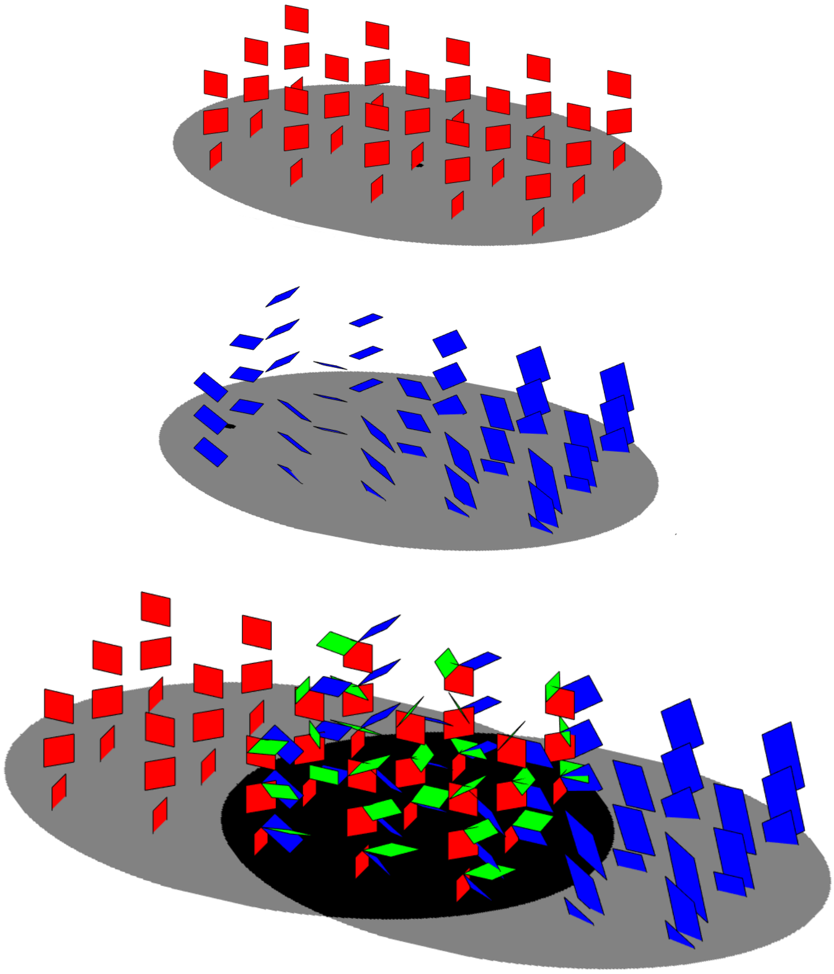

Figure 4: Lifting of the assemblage. Top: Lifting of the operator A_{p_0}. Middle: Lifting of the operator A_{p_1}. Bottom: Lifting of the assemblage $\&A_{p_0,p_1}$. Notice that the lifting of the assemblage is performed by the generators induced by A_{p_0}, the generators induced by A_{p_1} and their commutators (in green). Then the lifting of the assemblage is more than the union of the separated liftings, due to the presence of new commutators (difference of differences in the language of Gilles Deleuze). This assembly of planes indicates the possible directions of flows.

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In the intersection of the two bases we can as well compute the composition of the two operators: $\tilde{A}_{p_0} \circ \tilde{A}_{p_1}$. A more general version of the assemblage operator will contains also terms of this type.

More generally, if the bases do not intersect, we can define a propagation between them if there exist a chain of neighborhoods which connects them. Calling p_0 and p_k the two given points, this means that there exist $k-1$ points denoted $p_1, \cdots p_{k-1}$ such that the neighborhood of each one intersects the following one. In each intersection the previous process is applied.

The inverse of the assemblage operator is the disjunction operator: $(\&)^{-1}A$, that is able to generate two distinct operators A_{p_0} A_{p_1} starting from an integrate assemblage A.

3.3 The flow of the assemblage

As we have seen above, differential becoming is the flow u that is solution of an equation assciated to the assemblage operator A:

$$f(\partial_t u, \& Au) = 0.$$

In addition the function u will take values in a space H which will take into account material attributes, and it is allowed to change with rules similar to the ones described for the domain. We will also assume that $A(u)$ takes values in the same set H.

The space domain (B, F) of the solution is given a posteriori with respect to the definition of operators. If the concatenation changes, the space changes accordingly, giving rise to a morphogenesis of spaces.

The flux has values in a space of matters H, so that in the space domain we find formed substances with a density changing point to point. The flow appears to be as a cloud of formed substances continuosly changing in form, density, composition and velocity.

4 Genesis of the semiotic function

We will show in this chapter how the heterogentic flow and its polarization in principal axis will be at the base of the constitution of the semiotic function. These axis of cohesion will construct the expression / content planes in the sense of a generalized semiotics, not necessarly related to semio-linguistic contexte.

4.1 The semiotic function

The semiotic function, this co-existence of the sensible and the intelligible, is a blind spot of semio-linguistic knowledge. This is so de facto, but also probably de jure.

In the first place, this is so de facto as it is shown by the place that semiotic function occupies in the most eminent theoretical devices or the different treatments it receives.

Thus, by way of example, the glossematic theory of Hjelmslev which, in so far as it situates the interdependence between the planes (of expression and of content) above the articulation between form and substance at the level of which semio-linguistic knowledge is elaborated, thus excludes it on principle. Deleuze and Guattari in Thousand Plateau evoque exactly this hjelmslevian construction of a stratification of planes of expression and content.

Saussure, on the other hand, "delocalises" the question of semiotic function, but without succeeding. At first, he rejects the "grossly missleading" which is "(...) to consider a term as simply the union of a certain sound with a certain concept" ([44], pag 113), for it is the system that is first and the sign is only a "side effect". But in so doing, the initial gnoseological obstruction is only transposed, for henceforth it is the principle of a covering of the "horizontal" relations between signs (as relational units) and "vertical" (between signifier and signified) which makes problem. We know (see [22] pag. 238) that this problematization finds its expression culminated in a theory of value, but with no probative outcome.

The phenomenological perspective is not left out: the analysis of the sign developed by Husserl [26] met with the same obstacles, without overcoming them, since, the problem of the indivisible unity of the sign finding no internal answer, is in fine through the external superstructure an attentional consciousness field that the signifier and the signified, in so far as they occupy there specific positions, recover their unity [27].

These three approaches have in common the avoidance of a frontal examination of the semiotic fact either by overcoming it (Husserl), by remitting it (Hjelmslev), or by recomposing it (Saussure). More recently, and among others, linguistics of formalist or cognitive obedience dig this same path: whether it be a conventional correspondence between symbols and objects (model theory) or a dynamic convergence inscribed in an attractor, in each case the unity of the signifier and the signified is not reflected in its internal reason but reproduced by assembly - hence escaped in its own form.

4.2 Co-constitution of sensibility and meaning

But, in the second place, it is also so de jure. Indeed, as Hjelmslev implicitely considers it, semiotic function is not a phenomenon in the sense of empirical knowledge: the semiotic function does not allow itself to be apprehended in the way of a substance whose form it would be necessary to unveil or to bring out the laws which regulate their manifest functioning: its intelligibility is of another order.

Correlatively, it is the idea of a "crystalline" form, a system of explicit and univocal qualifications, without faults or opacities, in which the "almost" has no place, and able to produce in all clearness the truth of an object in itself fully determined, which is questioned. It is therefore before every form and substance, and therefore before the schemes of the empirical rationality correlative of the a priori of form and substance, that the proper reason of the semiotic fact must be sought. This is, in any case, the problematic line to which Mereleau-Ponty invites us, which therefore envisages the cross-constitution of a body and a world, both resulting from a play of interactions, in which the body, initially posed as a muffled vital power, and responding to the uncertain solicitations of a milieu that appeals to it, instructs it in return of its own rhythms, its specific behaviors, in order to then install in its outside a world of sensitive qualities.

In this movement of co-constitution, sensible qualities are, by construction, intrinsically signifying: the sensible is from the beginning provided with a meaning: that assigned to it by the corporeal matrix which institutes it. And the world in its native form is a world of expressions, in short a semiotically formed world.

4.3 The necessity of a genesis

The obstruction confronting semiotic thought would thus proceed from the fact that it posits the plans of expression and content on an epistemic stage in which they find themselves in the forms of empirical knowledge. When, then, each of them is provided with proper substances, their unity becomes unthinkable. In order to overcome this difficulty, it is necessary to locate its source, and to do this, to explain the basis of the articulation between form and substance, in other words the presuppositions of such an articulation. The Hjelmslevian system gives us the necessary concepts.

First there is the form: an ideal structure, precisely an abstract network of dependencies. But this form is incarnated, manifested; and this is precisely what the concept of substance relates. A third term is therefore necessary, matter, which relates the various amorphous which the form, by projecting on it, therefore produces in substance. In the glossematic, matter is precisely defined for the most part as an amorphous aggregate of unitary and independent atoms. But in so defining matter, Hjelmslev places it at the frontiers of the knowable.

On the one hand, matter is situated outside the realm of knowledge, simply because knowledge is concerned only with the relations of "cohesion" (see [23] pag.107), which the units of matter do not contract. But, on the other hand, matter is nonetheless conceptualizable, for, being therefore liable to "receive" forms, it must indeed have qualities that ensure its reception. Thus, even if it is devoid of form, matter is minimally formed (as a set of mutually univocal atoms) so as to be homogeneous to the forms and to constitute the ground of their possible actualizations.

Having these problematic elements, one returns better equipped to the examination of the semiotic function. Indeed, and in the first place as regards substances as such, there is therefore their implausible and unthinkable consubstantiality. Taking then the question according to the terms which factorize the substance, namely form and matter, two kinds of difficulties, all convergent, emerge.

First, as has already been mentioned, those which proceed from the gnoseological ideals of an objective knowledge restoring, under explicit and univocal properties, the totality of their object. But, as has also been mentioned, semio-linguistic facts do not lend themselves immediately to this kind of determination: for they appear primordially under the figure of the vague, the undecided, the "to be determined", and also the evanescent or stealth. Thus, it is the conception of a semiotic intelligibility which has been rendered and understood according to rigorous forms: limpid and systematic, which is to be reconsidered here. As far as matter is concerned, the obstructions are of the same nature: since it is conceived as a constellation of entities in themselves univocal, matter is immediately configured as "homogeneous", in that all its elements share a common nature which , undoubtedly makes them indifferentiable, but which dually binds them into a unitary and coherent mass. But once again, we have to question this formal a priori, because it is a myriad of mutually irreducible, singular and unskilled solicitations, even minimally, which are originally offered to our vital behaviors - solicitations which therefore fall within a field of existence in which the determinations are not yet acquired, and to which the very minimal form of homogeneity can not be without abuse attributed. It is therefore too much assigning them to conceive them in the form of simple units thus giving body to a homogeneous matter.

Thus, whether it be the originary installation of a signifying world in relation to a proper body (Merleau Ponty) or the primordial fact of an interpenetration of the planes of expression (Hjelmslev-Deleuze) and of content, it is to each time below all constituents or dimensions constituted that we must look for the elements of an explanation: therefore, especially below univocal and explicit formal regimes, but especially below the hypothesis of first units making "homogeneous" matter "and being a potential support for a more cohesive form - this for Hjelmslev-, and for Merleau-Ponty: below the stable and determined sensible qualities.

The examination of the fundamental forms of semiotics must therefore be initiated at this level in which a multitude of local tensions, mutually irreducible in the sense that they do not create common material, constitutes the primordial environment, by a sort of tightening towards the constitution of flows or aggregates, and further, the intrinsic constitution of principal dimensions, can be envisaged and studied.

4.4 The heterogenetic flow

To give ourself the ways, we reconsider now the multitude of heterogeneous singular differential constraints we have previously introduced, mutually inflexible operators, who, in what they are for each defined locally and concentrate, in their intensive sense, universes of possible forms, tell the main part of the " miscellaneous native of local tensions " previously envisaged.

Mathematical tools are then available to understand how this radically heterogeneous miscellaneous can, in echo in the updating of forms crossing it, be overtaken to the advantage of a kind of "weaving": where these tensions, at first mutually foreign, come into contact.

The existential scenario which is based on these supports is the following: in the beginning we will consider a "pulverulence" of local tensions, rendered through differential operators, a sort of nodes with intensive values in which the passages to an extensive actuality are played out, and which, as it were, seek to pass to existence.

Such an extensive existence, which as we have just seen, is correlative to the constitution and actualization of assemblages of operators, can be seen as the installation of a generalized vital flux , preceding all interiority or exteriority, and as such anterior to all body / world distinction.

The flux has all the characteristic of a morphological field with an internal consistency, since it is the integration of a differential problem. Then it tends to create coherent forms, but these are continuosly changing and are never stabilized in true gestalten. The flux is at the base of a complex theory of becoming that Deleuze invite to practice at a social, psychic, neurophysical, artistic and mathematical level.

The heterogenetic dispositive introduced so far poses the conditions for the differential becoming of a matter that is modulated by differential heterogeneous constraints to give rise to consistent configurations that never stabilise. The plastic composition of assemblages and their actualization induce to an incessant morfogenesis of formed substances.

It is in a later time that in this flux, which is thus a current response to the various intensive local tensions, and with regard to its history, that one can distinguish, on the principle of a harmonic analysis , internal dimensions of aggregation and closure on oneself.

4.5 Morphogenetic vibration

In his book on Francis Bacon [14], Deleuze writes :

“Sensibility is vibration. We know that the egg reveals just this state of the body “before” organic representation: axes and vectors„ gradients, zones, cinematic movements and dynamic tendencies, in relation to which forms are contingent and accessory”

and more

“ This is why we treat the Body without organ (the heterogenetic flux) as the full egg before the extension of the organism and the organisation of the organs, before the formation of the strata”. ‘ Franco Berardi commenting this passage in “And: Phenomenology of the end” [2] writes:

“Like a thin film recording and decifering non-verbal impressions, sensibility allows human beings to join together ... and regress to a non-specified and non-codified state of bodies without organs that pulsate in unison. “ “Sensibility is the faculty to decoding intensity, which by definition means to escape the extensive dimension of verbal language.Sensibility is the ability to understand the unspoken”.

In this perspective are the natural vibrations of the heterogeneous flow to determine the principal axes of decoding that constitute a first internal reference system without the need of any external decoding. Reading literally the idea of flow vibration suggested by Deleuze, these axes correspond to the principal or statistically independent components that are produced as vibrations of the process.

They will form a harmonic embedding of the process itself. If we define with $\&A(u)$ the concatenation of singular differential operatos, the embedding of the heterogeneous process will be defined by all the solutions of the spectral problem:

$$\&A_{p_0,p_1,\cdots,p_k}(u_i) = \lambda_i u_i$$

where u_i are the modes of vibration proper to concatenation, also known as eigenvectors. It is therefore within the heterogeneous process itself, the choice of the reference system in which to represent its evolution.

The instantaneous projection of the flow into its harmonic embedding is a point and its evolution is a trajectory in space. Sensible perception will therefore be the path of the flow in its harmonic embedding, even in absence of any stabilisation in fixed forms.

In [40] authors have shown that such a harmonic approach is able to individuate perceptual forms from visual stimuli. Extending this approach, in [42] visual plastic formants have been individuated, showing that the principal axes determine the reference system of the space where visual semiotic will develop later.

Hence, heterogeneous flow eigenvectors have a dual status. They are intrinsic reference flow axes and are continously varying forms, prepatterning of a possible successive stratification.

The dimensions thus revealed, carried by principal vectors, are at this stage unformed, and if we consider them from a merleau-pontian perspective, or that of a Deleuzian generalized semiotics, are nothing more than "possibilities", or of resources, of bodies and of the world, or of the possibilities of plans as expression and content, in their multiple stratification.

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