Uninvadable social behaviors and preferences in group-structured populations

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1 Introduction

- This talk is based on joint work with:
  
  - Jörgen Weibull (Stockholm School of Economics and Institute for
    Advanced Study in Toulouse)
  
  - Laurent Lehmann (Lausanne University, Department of Biology)

- We have so far written two manuscripts:

1. Lehmann, Alger & Weibull (2015): "Does evolution lead to maximizing
   behavior?" *Evolution*

   preferences in group-structured populations" WP
• Economists traditionally assume selfishness

• Sometimes “social” or “other-regarding” preferences are assumed: altruism (Becker 1974), warm glow (Andreoni 1990), reciprocal altruism (Levine 1998), inequity aversion (Fehr and Schmidt 1999), efficiency concerns (Charness and Rabin 2003), image concerns (Bénabou and Tirole 2006), a taste for truth-telling (Alger and Renault 2006)

• Some economists have included moral values in human motivation:
  – Smith (1759) and Edgeworth (1881)
  – Brekke, Kverndokk and Nyborg (2003), Bénabou and Tirole (2011)
• Study evolutionary foundations of human motivation!

• Evolutionary logic:

1. Human populations have evolved under scarcity of resources

2. Not all who are born survive and not all who survive reproduce

3. Darwinian logic: those alive today had ancestors who were successful at surviving and reproducing; our preferences should reflect this

• Goal: understand how the *environment* affects preferences
• Focus today on the propensity of our ancestors (last two millions of years) to live in small groups (5-150 grown-ups), extending beyond the nuclear family (Grueter, Chapais, and Zinner, 2012, Malone, Fuentes, and White, 2012, van Schaik, 2016, Layton et al., 2012)

• Part of the environment of evolutionary adaptation of the human lineage (e.g., van Schaik, 2016)

• Which impact may such group structure be expected to have on the long-term evolution of preferences?
• Population dynamics in group-structured populations:
  
  – a long-standing tradition in biology (Wright, 1931)
  
  
In the biology literature:
1. Results in terms of vital rates (fecundity, mortality, etc)
2. Almost exclusively on strategy evolution
Our contributions:
1. Results in terms of material payoffs
2. Allow for preferences to guide the strategy choice
2 Questions

- Imagine:

1. a large population divided into a large number of *islands*; each island has *n* slots

2. evolution takes place perpetually over discrete time; in each *demographic time period*:
   
   (a) a *material game* $G = \langle n, X, \pi \rangle$ is played in each island, where $\pi$ is the material payoff function

   (b) a certain number of transmission events occur (births, deaths, cultural transmission, migration, natural disasters, wars, etc.)

   (c) each individual achieves a certain *fitness* (defined as the expected number of *immediate descendants*)

- Under strategy evolution: each individual is programmed to play a certain strategy, inherited from its parent (biological or cultural)
Suppose that initially all individuals in the whole population plays the same strategy (the *resident* strategy) and that suddenly exactly one individual is born with a another, *mutant* strategy:

1. Does there exist any *uninvadable* strategy?

2. If so, is this strategy as if individuals sought to maximize some function? If so, which one?

Main result: Any uninvadable strategy can be interpreted as emanating from maximization of a utility function which involves selfishness, morality, and spite, at the level of material payoffs.
3 Examples

3.1 Example A: genetic transmission

- suppose the strategy is determined genetically, and that fitness is:

\[ w(\pi_i, \pi_{-i}, \bar{\pi}^*) = s(\pi_i) + m \cdot [1 - s(\bar{\pi}^*)] n \cdot \frac{f(\pi_i)}{n f(\bar{\pi}^*)} \]

\[ + (1 - m) \cdot \left( n - \sum_{j=1}^{n} s(\pi_j) \right) \cdot \frac{f(\pi_i)}{(1 - m) \sum_{j=1}^{n} f(\pi_j) + n m f(\bar{\pi}^*)} \]

where \( m > 0 \) is the probability for each newborn to migrate to another island, \( s(\pi_i) \) is the probability that \( i \) survives to the next demographic time period, and \( f(\pi_i) > 0 \) is \( i \)'s expected number of offspring.
3.2 Example B: genetic transmission and wars

- add potential wars between islands to the baseline scenario, and (for simplicity) set \( s(\pi_i) = 0 \) for all \( \pi_i \):

\[
w(\pi_i, \pi_{-i}, \pi^*) = [(1 - \rho) + 2\rho \nu(\pi, \pi^*)] \cdot \left[ m \cdot \frac{f(\pi_i)}{f(\pi^*)} + (1 - m) n \cdot \frac{f(\pi_i)}{(1 - m) \sum_{j=1}^{n} f(\pi_j) + nm f(\pi^*)} \right]
\]

where \( \rho \) is the probability that any given island is drawn into war, and \( \nu(\pi, \pi^*) \) is the probability that an island, in which material payoff profile \( \pi \in \mathbb{R}^n \) obtains, wins a war when the average payoff in the rest of the population is \( \pi^* \).
3.3 Example C: cultural transmission

- in every demographic time period, each adult dies and is replaced by exactly one child:
  - with probability $s(\pi_i)$: $i$’s child emulates $i$’s trait
  - with probability $[1 - s(\pi_i)](1 - m)$: $i$’s child emulates the trait of one randomly drawn grown-up in its natal island
  - with probability $[1 - s(\pi_i)]m > 0$: $i$’s child draws a sample of $n$ grown-ups from the population at large, and emulates the trait of one of them

$$w(\pi_i, \pi_{-i}, \bar{\pi}^*) = s(\pi_i) +$$

$$+ (1 - m) \cdot \left( n - \sum_{j=1}^{n} s(\pi_j) \right) \cdot \frac{f(\pi_i)}{\sum_{j=1}^{n} f(\pi_j)}$$

$$+ m \cdot [1 - s(\bar{\pi}^*)] \cdot \frac{f(\pi_i)}{f(\bar{\pi}^*)}$$

where $f(\pi_i) > 0$ is the attractiveness of the trait used by $i$. 
4 Answers

4.1 A first step: a condition for uninvasability

- Suppose that initially everybody has strategy $x$, and that suddenly one individual on an island switches to a mutant strategy $y$

- $x$ is uninvadable if $y$ is bound to disappear from the population in finite time
Proposition  A strategy $x$ is uninvadable if and only if

$$W(y, x) \leq W(x, x) \quad \forall y \in X.$$ 

- $W(y, x)$ is the \textit{lineage fitness} of the mutant strategy $y$:

$$W(y, x) = \sum_{k=0}^{n-1} p_k(y, x) \cdot w(\pi(y|k), \langle \pi(y|k), \pi(x|k) \rangle, \pi^*(x))$$

- An individual’s \textit{lineage}: the collection of this individual’s all descendants (her immediate descendants, the immediate descendants of her immediate descendants, etc. \textit{ad infinitum})

- $p_k(y, x)$ is the probability of a (uniformly) randomly drawn member of a local lineage to coexist with $k$ other members of the same local lineage
4.2 A second step: a condition for uninvadability under “weak selection”

- In biology “weak selection” refers to situations in which the fitness effects of heritable traits are small and have only first-order effects.

- Here, the assumption is that the material payoffs have only first-order effects on an individual’s fitness.

- Key implication: \( p_k(y, x) \rightarrow p_k^0 \quad \forall x, y \in X, \ k = 0, 1, \ldots, n - 1 \)
Proposition  

*Under weak selection, a strategy* \( x \in X \) *is uninvadable iff*

\[
\Pi(y, x) \leq \Pi(x, x) \quad \forall y \in X.
\]

- \( \Pi(y, x) \) is the mutant strategy’s *lineage payoff-advantage*:

\[
\Pi(y, x) = \sum_{k=0}^{n-1} p_k^0 \cdot \tilde{\pi}(k)(y, x)
\]

where

\[
\tilde{\pi}(k)(y, x) = \pi(y|k) - \lambda_0 \cdot \left[ \frac{k}{n-1} \pi(y|k) + \frac{n-1-k}{n-1} \pi(x|k) \right]
\]

and \( \lambda_0 \) is the coefficient of *local competitiveness*:

\[
\lambda_0 = - \frac{(n-1) w_n(\pi_i, \pi_{-i}, \bar{\pi}^*)}{w_1(\pi_i, \pi_{-i}, \bar{\pi}^*)} \bigg|_{\delta=0}
\]
4.3 Nash equilibrium

- Let $p^0 = (p^0_0, p^0_1, \ldots, p^0_{n-1})$ and consider the goal function defined by

$$u^0(x_i, x_{-i}) = \mathbb{E}_{p^0} \left[ \pi(x_i, \tilde{x}_{-i}) - \lambda_0 \cdot \sum_{j \neq i} \pi(\tilde{x}_j, \tilde{x}_{-j}) \mid x \right] \quad \forall x \in X^n$$

Corollary A strategy $\hat{x} \in X$ is uninvadable under weak selection iff it is a symmetric Nash equilibrium strategy in the $n$-player game in which every player’s strategy set is $X$ and every player has payoff function $u^0$.

- For $n = 2$:

$$u^0(x_i, x_j) = p^0_0 \cdot [\pi(x_i, x_j) - \lambda_0 \cdot \pi(x_j, x_i)] + p^0_1 (1 - \lambda_0) \cdot \pi(x_i, x_i)$$

- We call such individuals competitive moralists.
4.4 First-order condition for uninvadability

- The *pairwise relatedness coefficient* is the expected share of other lineage members among one's \((n - 1)\) neighbors:

\[
    r_0 = \sum_{k=0}^{n-1} \frac{k}{n-1} p_k^0
\]

**Corollary** Let \(\pi : X^n \to \mathbb{R}\) be continuously differentiable with \(X \subseteq \mathbb{R}\). If a strategy \(\hat{x} \in \text{int}(X)\) is uninvadable under weak selection, then

\[
    \left[ \pi_1 (y, \hat{x}) + (n - 1) \kappa_0 \cdot \pi_n (y, y) \right]_{y=\hat{x}} = 0,
\]

where \(\hat{x}\) is the \((n - 1)\)-dimensional vector whose components all are \(\hat{x}\), and \(y\) is the \((n - 1)\)-dimensional vector whose components all are \(y\), and

\[
    \kappa_0 = \frac{(n - 1) r_0 - \lambda_0 [1 + (n - 2) r_0]}{(n - 1) (1 - \lambda_0 r_0)}
\]
4.5 Example A: genetic transmission

\[
\begin{align*}
    w(\pi_i, \pi_{-i}, \bar{\pi}) &= s(\pi_i) + m \cdot [1 - s(\bar{\pi})] \cdot \frac{n \cdot f(\pi_i)}{nf(\bar{\pi})} \\
    &+ (1 - m) \cdot \left( n - \sum_{j=1}^{n} s(\pi_j) \right) \cdot \frac{f(\pi_i)}{(1 - m) \sum_{j=1}^{n} f(\pi_j) + nm f(\bar{\pi})}
\end{align*}
\]

Suppose that \( s(\pi_i) = s_0 \) and

\[
f(\pi_i) = f_0 \cdot \exp(\delta \cdot \pi_i)\]
Then:

\[
    r_0^A = \frac{(1 - m)^2 + (1 + m^2) s_0}{n - (n - 1)(1 - m)^2 + (1 - (n - 1)m^2) s_0}
\]

\[
    \lambda_0^A = \frac{(n - 1)(1 - m)^2}{n - (1 - m)^2}
\]

\[
    \kappa_0^A = \frac{2(1 - m) s_0}{2(1 - m) s_0 + n [2 - m (1 - s_0)]}
\]
Black solid: $s_0 = 1/n$ and $n = 2$
Black dashed: $s_0 = 1/n$ and $n = 10$
Blue solid: $s_0 = 0.8$ and $n = 2$
Blue dashed: $s_0 = 0.8$ and $n = 10$
Pink: $s_0 = 0$
4.6 Example B: genetic transmission and wars

\[ w(\pi_i, \pi_{-i}, \bar{\pi}^*) = [(1 - \rho) + 2\rho v(\pi, \bar{\pi}^*)] \cdot \left[ m \cdot \frac{f(\pi_i)}{f(\bar{\pi}^*)} + \frac{(1 - m) n \cdot f(\pi_i)}{(1 - m) \sum_{j=1}^{n} f(\pi_j) + nm f(\bar{\pi}^*)} \right] \]

Suppose that \( f(\pi_i) = f_0 \cdot \exp(\delta \cdot \pi_i) \) (as in the preceding example) and

\[ v_n(\pi, \bar{\pi}^*) = \frac{\exp(\delta \cdot n \bar{\pi})}{\exp(\delta \cdot n \bar{\pi}) + \exp(\delta \cdot n \bar{\pi}^*)} \]
Then:

\[ r_0^B = \frac{(1 - m)^2}{n - (n - 1)(1 - m)^2} \]

\[ \chi_0^B = \frac{(n - 1)(1 - m)^2 - \rho (n - 1) n/2}{n - (1 - m)^2 + \rho n/2} \]

\[ \kappa_0^B = \frac{\rho}{\rho + 2m(2 - m)} \]
Pink: $\rho = 0$  Orange: $\rho = 0.4$  Blue: $\rho = 0.8$
4.7 Example C: cultural transmission

\[ w(\pi_i, \pi_{-i}, \pi^*) = s(\pi_i) + \]
\[ + (1 - m) \cdot \left( n - \sum_{j=1}^{n} s(\pi_j) \right) \cdot \frac{f(\pi_i)}{\sum_{j=1}^{n} f(\pi_j)} \]
\[ + m \cdot [1 - s(\pi^*)] \cdot \frac{f(\pi_i)}{f(\pi^*)}, \]

Suppose that \( f(\pi_i) = f_0 \cdot \exp(\delta \cdot \pi_i) \) and \( s(\pi_i) = s \).
Then:

\[
C_0 = \frac{(1 - m)(2s_0 + (1 - m)(1 - s_0))}{n(1 + s_0) - (1 - m)(n - 1)(2s_0 + (1 - m)(1 - s_0))}
\]

\[
\lambda^C_0 = \frac{(n - 1)(1 - m)}{n - (1 - m)}
\]

\[
\kappa^C_0 = -\frac{(1 - m)(1 - s_0)}{2n - [m(n - 1) + 1](1 - s_0)}
\]
Pink: \( s_0 = 0, \ n = 2 \) \hspace{1cm} \text{Orange: } \ s_0 = 0.4, \ n = 2 \hspace{1cm} \text{Blue: } \ s_0 = 0.8, \ n = 2 \\
Pink \text{ dashed: } \ s_0 = 0, \ n = 10 \hspace{1cm} \text{Orange dashed: } \ s_0 = 0.4, \ n = 10 \\
Blue \text{ dashed: } \ s_0 = 0.8, \ n = 10
5 Concluding remarks

- Humans are social animals: many of our behaviors occur within small groups (the family, the neighborhood, schools, firms, economics departments, etc...).
- This has been the case for at least a couple million years.
- We show that evolution in such group-structured populations sustains *competitive moralists*, who are driven by a mixture of selfishness, spite (or altruism), and moral concerns.
- On balance, equilibrium behaviors may be anti- or pro-social, depending on whether the effect of local competition (which drives spite/altruism) outweighs or not the effect of relatedness (which drives the moral concerns).
- Moreover, for altruism to evolve, groups must have a common enemy (other groups, environmental shocks etc).
- Many questions remain to be answered: can heterogeneous population states be sustained, and if so, which?
Merci !