Frequency-Structure Decomposition of Link Streams
Big Picture

Graph decomposition coefficients

Edge space regions

Link stream edges

Time

Activity

Region index

Activity

Region index
$L(t, e) = 
\begin{array}{c}
\text{Edges} \\
\hline
f_{g_0}(e) \\
\hline
f_{g_1}(e) \\
\vdots
\end{array}
$


\[ L(t, e) = \left( \begin{array}{c|c|c|c|c} \text{Time} & \text{Regions} & \text{Edges} \\
\hline
\phi_0 & \phi_1 & \vdots \end{array} \right) \]

\[ = \begin{pmatrix} c_0(t) & c_1(t) & \ldots \end{pmatrix} \]
\[ L(t, e) = \begin{pmatrix}
\psi_0 & \psi_1 & \cdots \\
\end{pmatrix}
\begin{pmatrix}
\begin{array}{ccc}
\psi_0 & \psi_1 & \cdots \\
\end{array} \\
\end{pmatrix}
\begin{pmatrix}
\begin{array}{ccc}
\cdots & c_{01} & \cdots \\
\end{array} \\
\end{pmatrix}
\begin{pmatrix}
\begin{array}{ccc}
\cdots & c_{10} & \cdots \\
\end{array} \\
\end{pmatrix}
\begin{pmatrix}
\begin{array}{ccc}
\cdots & c_{ij} & \cdots \\
\end{array} \\
\end{pmatrix}
\begin{pmatrix}
\phi_0 \\
\phi_1 \\
\end{pmatrix}
\begin{pmatrix}
\end{pmatrix}
\]
Frequency-Structure Decomposition

\[ L(t, e) = \sum_{i,j} c_{ij} \psi_i \phi_j^T = \sum_{i,j} c_{ij} \zeta_{ij}(t, e) \]

- \( \zeta_{ij}(t, e) \): dictionary elements (structure \( j \) oscillating at frequency \( i \) )
Frequency and Structural Filters

Frequency and structural filters become easy to analyze

\[ \hat{L} = HLQ \]

- \( H \): frequency filter (matrix diagonalized by signal dictionary)
- \( Q \): structural filter (matrix diagonalized by graph dictionary)
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Example: Separating Link Streams

Stable cliques

Oscillating cliques

Random activity
Example: Separating Link Streams

- Low pass - coarse grain
- High pass - coarse grain
- All pass - details

Frequency-Structure Decomposition
Example: Backbone & Details

Frequency-Structure Decomposition
Example: Backbone & Details

[Diagrams showing frequency-structure decomposition over time, with specific emphasis on day and night activity patterns and structure frequencies.]
Example: Backbone & Details

Raw Link Stream

Snapshot

Reconstruction
6 highlighted coefficients

Snapshot
Example: Backbone & Details

Raw Link Stream

Snapshot

Reconstruction
100 largest coefficients

Snapshot
Example: Backbone & Details

Raw Link Stream

Snapshot

Reconstruction
1000 largest coefficients

Snapshot
Example: Backbone & Details

Raw Link Stream

Snapshot

Reconstruction
10000 largest coefficients

Snapshot
Example: Backbone & Details

Raw Link Stream

Snapshot

Reconstruction
all coefficients

Snapshot
Example: Backbone & Details

Raw data = Two daily active communities + details
Example: Backbone & Details

Same experiment: classical graph + signal decompositions
Data: Contact Network Lyon Hospital

Aggregated Network

Activity Time Series

- 29 Patients
- 27 Nurses
- 11 Medical Doctors
- 8 Administrative Staff
Application: Contact Network Lyon Hospital

Raw data: sample every hour

3 pm

4 pm

5 pm

6 pm

7 pm

8 pm

9 pm

10 pm

Patient  Nurse  Doctor  Admin

Frequency-Structure Decomposition
Application: Contact Network Lyon Hospital

Filtering out largest frequencies and details

3 pm

4 pm

5 pm

6 pm

7 pm

8 pm

9 pm

10 pm

Patient

Nurse

Doctor

Admin
Application: Contact Network Lyon Hospital

Comparison: tensor factorization (upper) / MDS-NMF (bottom)
Conclusions

- Proposed novel decomposition for link streams
  - Time axis: Handled by classic signal transforms
  - Structural axis: Handled by novel graph decomposition
  - Meaningful for any raw data
- Graph decomposition that interacts well with signal decompositions
  - Measures edge activity in regions of edge-space
  - Interesting properties to characterize and compare graphs
- Time and graph filters easily interpretable in frequency-structure
  - Operate together to suppress specific frequency-structure combinations
  - Allow to separate stable, oscillating and noisy activity
  - Useful to retrieve the backbone of a link stream