Deterministic preparation of a two-dimensional soliton in a scale-invariant Bose gas

Jérôme Beugnon

Laboratoire Kastler Brossel. Collège de France


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Matter-wave solitons

Bright solitons (attractive interactions)

Dark solitons (repulsive interactions)

Strecker et al. & Khaykovich et al. (2002)

Burger et al. (1999)

Multi-component solitons

Jones-Roberts soliton

Farolfi et al. & Chai et al. (2020)

Meyer et al. (2017)
Non-linear (cubic) Schrödinger equation

Cold gases are a good platform to explore non-linear physics

Mean-field regime, zero temperature $\rightarrow$ Equilibrium NLSE:

$$
\mu \phi(r) = -\frac{1}{2} \nabla^2 \phi(r) + gN|\phi|^2 \phi(r) \quad (\int |\phi|^2 = 1)
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- Choose dimension (2D in this work)
- Tune interaction strength $g$ from positive to negative values
- Several components ($\leftrightarrow$ coupled NLSE)
- Large control on design and measurement of the complex field $\phi$
Our setup: 2D Bose gases in box potentials

Full control on the density distribution

Spatial light modulator (DMD) ⇒ 2D box potential

\[ T \lesssim 50 \text{ nK}, \ n_{2D} \approx 100 \ \mu \text{m}^{-2} \Rightarrow \text{superfluid regime} \]

Ville et al. PRA 95, 013632 (2017), Zou et al. J. Phys. B 54 08LT01 (2021)
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One or Two-component system

\[ ^{87}\text{Rb} \]

\[ |F = 2, m = 0 \rangle \equiv |2\rangle \]

\[ |F = 1, m = 0 \rangle \equiv |1\rangle \]

density: \[ n_i = |\phi_i|^2 \]

\[
\begin{align*}
\mu_1 \phi_1 &= -\frac{1}{2} \nabla^2 \phi_1 + (g_{11} n_1 + g_{12} n_2) \phi_1 \\
\mu_2 \phi_2 &= -\frac{1}{2} \nabla^2 \phi_2 + (g_{12} n_1 + g_{22} n_2) \phi_2
\end{align*}
\]

All interactions parameters repulsive:

\[ g_{11} \approx g_{22} \approx g_{12} > 0 \]
A recent work: breather in a harmonic trap

Uniform cloud with equilateral triangular boundaries in harmonic trap of period $T$:

Saint-Jalm et al. PRX 09, 021035 (2019)
A recent work: breather in a harmonic trap

Uniform cloud with equilateral triangular boundaries in harmonic trap of period $T$:

Periodic oscillation with a period $T/2$

$\Rightarrow$ Non-trivial non-linear (many-body) effect
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Interpretation: Mapping to an ideal Fermi gas Shi et al. PRX 11, 041031 (2021)
Soliton formation:

Competition $E_{\text{kin}} \propto \int |\nabla \psi|^2$ vs $E_{\text{int}} \propto gN \int |\psi|^4$ (interaction parameter $g < 0$)

In 1D, wavepacket size $\ell$: $E_{\text{kin}} \propto 1/\ell^2$ and $E_{\text{int}} \propto gN/\ell \Rightarrow \ell_{\text{soliton}} \sim \frac{1}{|g|N}$
2D solitons of NLSE

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Stationary weakly unstable solutions only for discrete values of $\tilde{g}N$.

$|\tilde{g}|N = 5.85 \ldots \rightarrow$ Townes soliton $\phi_\mu(r) = R_\mu(r)$
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(Real, axially symmetric and monotonically decreasing as $r \to \infty$)

Non-linear optics: competition self-focusing vs diffraction (Chiao et al. PRL (1964))
Scale-invariance of 2D Bose gas

Equations of motion conserved in dilation of space $r \to r/\lambda$ and time $t \to t/\lambda^2$. Weakly-interacting Bose gas with contact interaction is scale-invariant

Consequences

- Interaction parameter $\tilde{g}$ is dimensionless
- Townes soliton has no imposed size: $\lambda R(\lambda r)$ is a solution of 2D-NLSE with $\mu \to \lambda^2 \mu$.
- Virial theorem (Pitaevskii & Rosch, PRA (1996)):
  
  $d^2 \langle r^2 \rangle dt^2 = 4E_m$ with $E = E_{\text{kin}} + E_{\text{int}}$

  If zero initial velocity:
  
  $\langle r^2(t) \rangle = \langle r^2(t=0) \rangle + 2E_m t^2$

  Stationary Townes solution $\Rightarrow E = 0$.

  Zero energy: necessary but not sufficient condition to have a soliton
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Effective attractive interactions in a two-comp. gas

Recent observation of Townes soliton in a single-comp. attractive gas.
(2D attractive Bose gas: Chen & Hung, PRL 2020, PRL 2021)
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**Our approach:**

Two-component gas with repulsive interaction parameters: $g_{11}$, $g_{22}$ and $g_{12} > 0$.

$$g_e = g_{22} - g_{12}^2 / g_{11}$$
Effective attractive interactions in a two-comp. gas

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(2D attractive Bose gas: Chen & Hung, PRL 2020, PRL 2021)

Our approach:
Two-component gas with repulsive interaction parameters: \(g_{11}, g_{22} \) and \(g_{12} < 0\).

Two impurity atoms in a bath

\[ g_e = g_{22} - \frac{g_{12}^2}{g_{11}} \]

\( g_e < 0 \iff \text{immiscibility criterion } (g_{22}g_{11} < g_{12}^2) \)

Impurities immiscible with their bath \(\iff\) attractive gas

No length scale from the bath \(\rightarrow\) scale-invariance (approximately) conserved in 2D
Optical imprinting of an arbitrary spin distribution

Raman transfer

$|F = 2, m = 0\rangle \equiv |2\rangle$

$|F = 1, m = 0\rangle \equiv |1\rangle$

Zou et al. J. Phys. B 54 08LT01 (2021)
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Preparation of a Townes soliton

Townes-shaped wave packet, RMS size $\sigma_0 = 5.7 \, \mu m$, $\tilde{g}_e \approx -7.6 \times 10^{-3}$
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Townes soliton expected for $N = N_T \equiv 5.85/|\tilde{g}_e| \approx 770$
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First deterministic realization of a Townes soliton!
Stationary atom number and scale-invariance

Virial theorem: $\langle r^2(t) \rangle = \sigma_0^2 + \left( \frac{\hbar}{m\sigma_0^2} \right)^2 \gamma t^2$ with $\langle r^2(t = 0) \rangle = \sigma_0^2$

Expansion coefficient: $\gamma = \alpha (1 - N/N_T)$ with $\alpha \approx 1$
Stationary atom number and scale-invariance

Virial theorem: \( \langle r^2(t) \rangle = \sigma_0^2 + \left( \frac{\hbar}{m\sigma_0^2} \right)^2 \gamma t^2 \) with \( \langle r^2(t = 0) \rangle = \sigma_0^2 \)

Expansion coefficient: \( \gamma = \alpha(1 - N/N_T) \) with \( \alpha \approx 1 \)

Data for all \( \sigma_0 \) collapse on a single curve with \( N_T \sim 800 \)

Signature of \textit{scale invariance}
Is having the Townes profile really important?

Compare with the case of a Gaussian

Zero-energy state expected for $\tilde{g}_e N_G = 2\pi$
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Initial Gaussian profile is strongly modified
Exact Townes shape is required
From Townes soliton to spin domains

Cubic NLSE only for $n_2 \ll n_1$. Exact Townes soliton appears in the limit $n_2 \to 0$.
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$\mu \psi = -\frac{1}{2} \nabla^2 \psi + \tilde{g} n \phi + \nabla^2 \sqrt{n_\infty - n_2} \sqrt{n_\infty - n_1} \phi$

$\mu$ stabilizes the wavepacket $\Rightarrow$ breaks scale-invariance (bath density: $n_\infty$) $\Rightarrow$ shifts the equilibrium atom number

Continuous connection from Townes soliton to spin domains.

Spin domains = full spatial separation between components 1 and 2, 100% depletion.

One can show that stationary state must have $N \geq N_T$ and $E \leq 0$. 

Zakharov et al. Sov. Phys. JETP, 1971
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For $g_{11} \sim g_{22} \sim g_{12}$:

Two coupled NLSE $\Rightarrow$ effective one-component NLSE

$$\mu_e \phi = -\frac{1}{2} \nabla^2 \phi + \tilde{g}_e n \phi + \frac{\nabla^2 \sqrt{n_\infty - n}}{2\sqrt{n_\infty - n}} \phi$$
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Excitation spectrum of spin bubbles

Townes soliton: no internal vibration modes
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Townes soliton: no internal vibration modes

\[ s = 0 \]

\[ 1 \quad 1.2 \quad 1.44 \]

\[ \omega_s / 2\pi \text{ (Hz)} \]

\[ N / N_T \]

Not bound
Excitation spectrum of spin bubbles

Townes soliton: no internal vibration modes

\[ s = 0 \]
\[ s = 2 \]

\[ \frac{\omega_s}{2\pi} \text{(Hz)} \]

\[ N/N_T \]

Not bound
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Townes soliton: no internal vibration modes

\( s = 0 \quad s = 2 \quad s = 3 \quad s = 4 \quad s = 5 \quad s = 6 \quad s = 7 \)

\( \omega_s / 2\pi \) (Hz)

\( N/N_T \)

Not bound

No internal mode
Conclusion and outlook

Deterministic preparation of a Townes soliton

Outlook:
- Moving solitons and collisions between solitons
- Higher-order 2D solitons
- Beyond classical-field limit (spatial-dependence of the interaction parameter, Hammer & Son PRL, 2004)
- Rogue waves, Peregrine soliton (Romero-Ros arXiv:2112.03845)

Contributors
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Two-component simulations

Ground state for increasing atom number (10, 1.5, 1.01 $N_T$)
Two-component simulations

(a) \( \frac{N_2}{N_T} \) vs. \( \mu_2/\mu_1 \) for different values of \( \mu_\text{min} \) and \( \mu_\text{max} \).

(b) \( \sigma/\ell_0 \) vs. \( N_2/N_T \) for different values of \( N_2/N_T \).

(c) \( \frac{n_2(0)}{n_\infty} \) vs. \( N_2/N_T \) for different values of \( n_2(0)/n_\infty \).

(d) \( 1 - Q_T \) vs. \( N_2/N_T \) for different values of \( 1 - Q_T \).