The time of populations: reconstruction, dynamics, and viability

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1 The time of populations

1.1 The unexpected

Part of demography is concerned with the study of stable populations, convergence toward equilibria, cycles, asymptotic distributions. However, human populations, by their historical nature, call for a different way of thinking about time. For example, fertility time series show clashes, breakdowns, regime shifts, temporary stagnation, unexpected unexpectedness. This variability of time signals raises the question of the viability of underlying systems: how, in our advanced economies, to manage the jolts of the age structure, such as the massive arrival of schoolchildren, then students, then job seekers, then pensioners now? How, in traditional populations, to maintain social cohesion over time and generations on a given territory, despite wars, epidemics, food shortages? What makes an innovative behavior such as doing fewer children appear and diffuse in the geographical and social space, as was the case in France from mid-eighteenth century and throughout the nineteenth?

These questions cover those of the non-linearity of processes in demography, economics, population genetics, those of uncertainty and predictability of systems, and those of human agency.

2 The time of demographic change

2.1 Reconstructions of past populations

Parish registers have made it possible to constitute series of marriages, baptisms, and burials. I proposed a method of reconstructing the population from long series of baptisms and burials,[1] based on a certain stability in the trend of life expectancy before and after the start date of the burial series.

In the presence of regular censuses, I proposed an original reconstruction capable, based on death by age data,[2] of correcting data from the *Statistique générale de la France* and resulting in a new view of demographic forces (fertility, mortality, net migration by age).

[1] N. Bonneuil and J. N. Biraben (1986) "Population et Société en Pays de Caux au XVIIe siècle," *Population* 6, 937-960.

[2] N. Bonneuil (1997) *Transformation of the French demographic landscape.* Oxford: Clarendon Press, 224 pages.

A difficulty often encountered in the analysis of historical data is that of flaws and recording errors. I proposed an original method combining stochastic optimization with Lotka-McKendrick equations with migration, where the distance to be minimized is the difference between reconstruction and vital statistics or censuses. With Elena Fursa, from the Federal University of South Russia, we dealt with the empirical case of administrative subdivisions in southern Russia between 1863 and 1916.[3]

The reconstruction of the marriage market and the time series of age pyramids by civil status from flawed registration is obtained as a solution to another minimization programme in large dimension. The correspondence of ages between brides and grooms at each month of the year between 1867 and 1916 is also obtained as a solution to a stochastic minimization program, avoiding the introduction of ad hoc marriage functions.[4]

2.2 Demographic paths in the distant past

An overview of the distant past of the populations is now made possible by the possibility of exploiting molecular genetic data. Demographic fluctuations in the distant past indeed are reflected by the heterogeneity of the genome, in particular that of mitochondrial DNA, which has a higher mutation rate than nuclear DNA. Does the distribution of nucleotide pairs bear the signature of a “bottleneck” that would have occurred thousands of years ago, as it has been extensively commented on? Montgomery Slatkin, in the laboratory where I worked for six months (Berkeley, California), showed that collected samples are consistent with the assumption of a population constant since the beginning, as they are also with that of an exponential population.

I offered to find[5] the set of all possible temporal paths capable of producing the observed genetic heterogeneity (polymorphism). It is a viability problem: I am looking for the largest set of states from which there is a solution leading to the observed result, which is here the confidence interval of the number of genetic differences within a sample of individuals (nucleotide differences within a random pair of genes). The solution I am talking about here is the so-called “coalescent” process: knowing the mitochondrial genomes of

[3]N. Bonneuil and E. Fursa, (2011) “Optimal Population Path Fitting for Flawed Vital Statistics and Censuses,” *Journal of Optimization Theory and Applications* 148-2, 301-317. Bonneuil, N. (2017) “Reconstruction of populations by stochastic optimization: sensitivity analysis,” *Mathematical Population Studies* 24(3), 181-189.

[4]N. Bonneuil and E. Fursa, (2012) “Optimal Marriage Fitting for Imperfect Statistics,” *Journal of Optimization Theory and Applications*, 153, 532-545.

[5]N. Bonneuil, (1998) “Population paths implied by the mean number of pairwise nucleotide differences among mitochondrial DNA sequences,” *Annals of Human Genetics* Jan 62, 61-73.

individuals in a sample, one calculates the probability that two of them have the same common ancestor a number of generations ago. This probability depends on the history of the population size. In addition, the genetic differences between these two individuals are due solely to mutations since the common ancestor. Hence the relationship between genetic differences and demographic fluctuations of the past, which nevertheless remains difficult to treat when the population is considered variable over time.

Rather than validating a plausible or spectacular scenario (such as the famous demographic "bottleneck"), my answer is: here is the domain of possible stories reflected in the data (this domain can sometimes be reduced to a single one trajectory). The difficulty of such an attitude is due to the mathematical apparatus, which is not immediate, and in the case of the theory of viability and set-valued analysis, is unusual.

2.3 "Paleo-demography"

Another case study comes from paleo-demography: an attempt has been made to deduce fertility and mortality from the age distribution (whose very determination is questionable) of skeletons found in cemeteries. The simplest scenarios, such as stationary or stable population models, give a relatively poor fit to the data. It is unlikely that such simple demographic paths have ever existed, if we bear in mind the very irregular fluctuations in fertility and mortality reconstructed in the 17th century in France. On the contrary, my position was to point out that the age distribution of skeletons only gives an indication of all demographic trajectories, those passing through the demographic states of the capture domain of this distribution, which then appears as a target in an appropriate state space, under a Lotka-McKendrick type dynamic. Of these demographic paths producing the recorded distribution of deaths by age, one is the most parsimonious in terms of fluctuations in fertility and mortality, and deviation from the stable age structure. Finding it requires a radical increase in the number of degrees of freedom (from 2 in the stable case to 25 in the case of Belleville Cemetery, Ontario). This is what I have implemented by relying on modern techniques of stochastic optimization.[6] For example, I have shown that the average life expectancy and average fertility are thus correctly reconstructed, and I have been able to estimate these measures in the cases of Belleville Cemetery, Ontario (1821-74) and Dallas, Texas (1869-1907).

[6] N. Bonneuil (2005) "Fitting to a distribution of deaths by age with application to paleodemography," *Current Anthropology* 46, 29-45.

2.4 The fluctuations of fertility in the old demographic regime

2.4.1 Territorial regulation in Pays de Caux, 1588-1700

The French 17th century is an exceptional epoch for observing populations subjected to drastic conditions of mortality and subsistence. The plague occurred every thirty years, when this was not war, dysentery, or famine. Observed series (baptism, marriages, burials) have a very irregular appearance.

To "make these data speak," some historians identify "higher" peaks in the number of deaths, which they call "crises of mortality". Others are content with studying fluctuations in short-term using simple econometric models.[7]

It is more difficult to rebuild past populations, to reveal processes of mortality and fertility. In Pays de Caux, 1588-1700, censuses do not exist and I proposed an original method based on the only parish series data to obtain series of fertility and mortality indicators over these 113 years of the old demographic regime.

Their extremely irregular appearance discourages one from understanding the underlying dynamics, if we stick to traditional demographics or econometrics. Replacing these trajectories in an appropriate mathematical space (the state space) restores the clashes, jolts, slow or rapid phases. This operation clinically delivers the anatomy of crises, the contextual ups and downs. A model combining land use constraints, nuptiality and fertility with mortality rates is a good explanatory candidate, capable not only of fitting for all reconstructed series, but also of tracing the series of recorded numbers of marriages, a series hitherto independent of the reconstruction and simulation.

The fertility of this population is based on two levels, made concrete by two attractors exchanging the trajectory: a low level corresponding to an ageing population due to the absence of a recent crisis, and a high level, corresponding to the arrival of young couples on lands freed from their former occupants by a recent mortality crisis. The age pyramid of marriable people also plays a role: aged by the absence of crises, it is able to cushion losses when a crisis occurs, maintaining fertility at a low level; decimated by successive crises, it provides a large contingent of young married couples, so eventually a high fertility. Finally, this model derived from the empirical data of the Pays de Caux allows me to deepen the consequences of such a relationship between populations and resources. By proceeding by simulation and by varying the probability of occurrence of mortality crises, I have shown

[7] N. Bonneuil (1991) "Temporalités en démographie historique," *Histoire et Mesure* VI-1/2, 137-148.

that there is a bifurcation value, that is, a value on either side of which the system operates in different regimes. Beyond eight crises per century, fertility remains high, maintained by high mortality, and the population, although fertile, is doomed to extinction if mortality prevails definitely on reproduction; below that value, fertility possesses two regimes, and this is the case of the empirical study of the Pays de Caux; in the absence of a crisis, fertility confines itself to a single regime, of low fertility, and corresponding to high ages at marriage. This study not only allowed me to validate the territorial regulation model on real historical data for the first time, but also to highlight its properties associated with historical mortality fluctuations.[8]

2.4.2 Malthus-Boserup

This case study raises the question of population interactions and environment. After Malthus denounced population growth as a source of impoverishment, Boserup argued that a large population has a positive effect on technological innovation. However, some empirical evidence points in Boserup's direction, others do not.

I proposed to revise the theoretical aspect of this debate by giving a central place to inertia (the time necessary for action to proceed) inherent in technological change, and considering innovation no longer as a mechanical process, but as a set-valued function circumscribing possible futures.[9] This makes it possible to find Boserup's ideas without postulating them, but also to restore the contingency and unpredictability in human behavior.

By representing the space of possibilities under constraints, I showed how the Boserupian hypothesis is incorporated into a broader perspective, where a myriad of Boserupian dynamics coexists with other non-Boserupian ones. When time passes and the population, growing, is approaching the Malthusian balance, synonymous with low income, the space for possibilities is shrinking, constituting an increasingly urgent exhortation to invent new techniques. But this is not mandatory, as the observation shows.[10]

[8] N. Bonneuil (1990) "Turbulent Dynamics in a 17th century population," *Mathematical Population Studies* 2, 289-311.

[9] writing a differential inclusion $x' \in F(x)$.

[10] N. Bonneuil (1994) "Malthus, Boserup and Population Viability," *Mathematical Population Studies* 4(5), 107-119.

2.4.3 Viability of traditional populations: nomads and fishermen

Anthropologist Fredrik Barth (1981) describes another traditional society, that of the Basseri nomads. He shows how this society was organized in such a way as to ensure its perpetuation, and identifies the processes by which the unity of the camp was maintained. When Barth uses the notion of equilibrium and static games to explain the history of the group, I showed you[11] that he is actually talking about dynamic games and viability out of equilibria. Equilibria, on the contrary, means famine, and the problem of nomads is precisely to stay away from this equilibrium as long as their resources and demographics allow. To achieve this, the group uses "strategies": successively the choice of the site where to settle temporarily (when should we leave the camp and where to go?), reducing individual consumption in a situation of food shortage, and finally sedentarization of the poorest, thus selling their labor force on farms, so as to relieve livestock from the burden of human demand. If all these regulations fail, mortality increases until the population is adjusted to resources. In this story, the nomads take decisions under the pressure of present and future constraints; they are therefore constantly changing their own history, their "trajectory" in the space of possible states. They cannot consider their future in terms of trajectories, but rather in terms of attainable sets, and the group leader is precisely the one in charge of making viable decisions, doing the best he can to avoid making a wrong decision; technically speaking, these decisions are the ones that lead the group within all the constraints of survival of the group (with the additional difficulty that interactions between humans and livestock are non-linear). In this concrete example, decision-making under uncertainty (itself due to the unpredictability inherent in land availability, meteorology, natural disasters, ...) is essential. Instead of a chronological time, we have a social time, where a finite number of controls generates a multitude of different stories.

This traditional population has therefore invented original preventive checks, exempting couples from having to use the most traditional delay in age at marriage or fertility restriction. It perpetually invents its own history, according to its resources and development.

In his other study of Norwegian fishermen, Barth shows that boat skippers must con-

[11] N. Bonneuil (1997) "Games, Equilibria, and Population Regulation under Viability Constraints: an interpretation of the work of the anthropologist Fredrik Barth," *Population English Selection* special issue New Methods in Demography, 151-179.

stantly choose between following other boats or go alone in search of new fishing banks. The strategy therefore changes all the time according to the behavior of the other boats. I have showed that it is in fact a dynamic problem, by describing the *capture domain* in the state space of capital (boat and crew) at time t, catch volume, and probability at t to find a better catch by taking risks. It is no longer a question of predicting what other actors will do, but to determine the set of states from which a skipper still has the possibility to avoid ruin, and what strategies, depending on the state of the system, will allow him to do so. How a skipper actually reacts, in a more or less skilful manner, produces a story; the capture domain and its associated set of viable strategies discriminate against success and bankruptcy among all the possible stories. As in this concrete case, the modern theory of dynamic games overcomes the limitations inherent in the static frame of game theory.

2.5 Diffusion of behavior and demographic transition

2.5.1 The French transition 1806-1906

Between the fluctuations at low average levels of fertility nowadays and those at high average levels in the 17th century, the "demographic transition" raises the question of the change in behaviors in time and space. To invest it, I have taken over the data from the *Statistique générale de la France* in the 19th century by *département*.

Frequent censuses (every 5 years) are detailed at the *départmental* level, and death statistics by age were held annually from 1851 onwards. These documents are, like many other statistics, flawed by counting errors and under- or misreporting. This issue inherent in population statistics should not be overlooked. The careful consideration of under-recording in the case of the 19th century allowed me to refer to the myth of the baby boom of the 1870s, and to reposition the French path which, although it remained a pioneer in Europe, was not as marginal as it has been claimed. This result, independently arrived at by the American researcher David Weir, can create an interesting debate, insofar as the belief in a French advance in the decline of fertility and its fluctuations in the 19th century has given rise to an abundant literature, both on the historical role of France and on the general rhythm of a demographic transition.

Based on my population reconstruction from 1806 to 1906 by *département*,[12] augmented

[12] N. Bonneuil (1997) *Transformation of the French demographic landscape.* Oxford : Clarendon Press, 224 pages.

with explanatory variables such as the degree of urbanization or the level of female education, the transition appears as a dynamic system in space and time. Appropriate econometric techniques (clusters, principal component analysis, simultaneous regressions, co-integration) make it possible to study it.[13] In particular, the urban hierarchy, which had been obscured due to a lack of adequate data reconstruction, is revealed fundamental to the transition. Finally, the very nature of this process is twofold: at the beginning of the transition, fertility behavior adapted to a changing environment and, once initiated, declined further at its own pace, which was similar to a wave of innovation travelling through space from diffusing centres.

2.5.2 Social mobility in 19th century France

I contributed to exploiting a nominative survey on the 19th century, the so-called "TRA" by Jacques Dupâquier and Denis Kessler, bringing together nearly 45,000 acts of marriage belonging to 3000 genealogies followed over time. Here again, what interests me is to study the change in social and demographic behaviors.

In particular, the TRA data are well suited for the analysis of social mobility. Experts in this field build transition matrices, where fathers and sons are classified according to pre-established categories. However, the occupation label at that time overlaps with a definition depending on time and place. So I proposed to build a continuous score to be assigned to an occupation neutralizing the effects of place and time. Fathers and sons or mothers and girls then become comparable without predefined categories. The father-son transformation is defined as a mathematical correspondence (which associates a set with a point). I characterized this correspondence by segmenting it with econometric models.[14] I was thus able to characterize the geographical space and the space in terms of social mobility, taking into account variables of education, occupation, residence, mortality of parents, and time.

[13] I have also, in another subject of study, proposed an original treatment of qualitative time series, as part of the study of the conflicts of 163 minority groups followed from 1945 to 1995: N. Bonneuil and N. Auriat (2000) "Fifty Years of Ethnic Conflict and Cohesion," *Journal of Peace Research* 37(5), 563-581.

[14] N. Bonneuil and P.A. Rosental (1999) "Changing social mobility in 19th century France," *Historical Methods* Spring 32-2, 53-73.

2.5.3 Migrations in the 19th century and the role of kinship

The genealogical linking of the marriage certificates of the TRA survey still opens up a unique opportunity to study the family components of migration. The family interdependence of behaviors prevents people from being treated as statistical observations independent of each other. On the contrary, I proposed to build a multi-level biographical analysis with simultaneous equations, where I could test the role of the family "network".[15] I confirmed the presence of two migration processes, short and long distance, the importance of the occupations of parents and step-parents, of the older brother or sister, the influence of the first and last generation, of education, . . .

2.5.4 The demographic transition in Egypt, 1960-1996

With Chouaaa Dassouki, I have also reconstructed fertility in Egypt from 1960 to 1996, on the one hand on the spatial scale of governorates divided into 38 rural and urban subdivisions covering the territory,[16], on the other hand, on the scale of *qaryas, medinas, shiyakhats, qisms*, which amount to 4905 administrative subdivisions.[17] By its geometry and its contrasting economic and political history, the case of Egypt is particularly interesting. Willing to study the relationship of reconstructed fertility series with those of mortality and migration, those of educational levels, marital status, and employment variables in different sectors encounters the difficulties that, during the demographic transition, many variables vary jointly and are autocorrelated in space and time. The model addresses these difficulties, and makes it possible to test the diffusion from the capitals of governorates, the large cities (Cairo, Alexandria, Port Saïd), or the nearest neghbors. One result is that, after the economic, health, educational, and family planning effects, there remains a strong diffusion of behaviors, contributing to a holistic portrait of the transition. Another result is that the speed of change in the 1970s created a strong heterogeneity within the same governorate: the transition did not affect behavior in a uniform way, but causes disparities that take time to vanish. The brief baby boom of 1974-85 suspended

[15] N. Bonneuil, Arnaud Bringé, and Paul-André Rosental (2008) "Familial Components of First Migrations after Marriage in nineteenth century France," *Social History* 33-1, 36-59.

[16] N. Bonneuil and C. Dassouki (2007) "Economics, Geography, Family Planning and Rapidity of Change in the Demographic Transition: the Case of the Egyptian Muhafazas 1960-1996," *Journal of Developing Areas* 40-2, 185-210.

[17] N. Bonneuil and Chouaa Dassouki (2006) "Women's Education and Diffusion of the Fertility Transition: the Case of Egypt 1960-1996 in 4905 Administrative Subdivisions," *Journal of Population Research* 23-1, 1-24.

these disparities for a period of time, especially in the Upper Valley, because fertility in the subdivisions advanced in the transition suddenly rises under the effect of an economic boom and a political seizure. The map of the disparities we have identified is a useful guide to implementing family planning.

2.5.5 The transition of contraception in Colombia

The analysis of multi-level multi-level multi-process biographies allowed me to identify the determinants of the contraceptive transition in Colombia between 1950 and 1994 for women born between 1935 and 1994 1979.[18] Individual trajectories show many discontinuities in contraceptive use. The 1935-39 cohort of rural women cohort shows the traditional behavior of not using contraceptives or using traditional methods (Ogino, interrupted intercourse, shower), while the 1970-79 cohort of urban women is the most likely to adopt contraception. Young women use it discontinuously, while a great proportion of fertility is concentrated at these young ages. Contraception is strongly associated with material living conditions, with education playing only a very secondary role (phenomenon noted in other studies). No preference for one or the other sex of the children appears; no significant difference either between town and country. The pill and the male condom are often used for a short period of time and in intermittently. The IUD and traditional methods (and, well sure, sterilization) are used for a longer period of time. The model is supported by qualitative interviews, which reveal the tension between the desire to fit into the traditional social norm of a large family and the desire to adopt a modern way of life corresponding to a smaller family.

2.5.6 The influence of economics on the demographic transition

On the theoretical level, I revisited the theory of the demographic transition based on the regulation of the economy of the family.[19] The consideration of mortality changes the equilibrium with unilateral gift. I expressed each spouse's allocation of the common good in accordance with the eminent role of mortality in the decline of fertility. The other determinant of equilibrium is the husband/wife productivity ratio. In a context of

[18] N. Bonneuil and Margarita Medina (2009) "The transition of contraception in Columbia 1950-1994," *Desarollo y sociedad* 64, 119-151.

[19] N. Bonneuil (2010) "Family Regulation as a Moving Target in the Demographic Transition," *Mathematical Social Sciences* 59, 239-248. N. Bonneuil (2017) "Maintain and acquire: the viability principles in population economics," *Revue d'économie politique* 127(2), 153-172.

demographic transition, the adjustment of allocations to their equilibrium values has no reason to be instantaneous, and the transition is more like the pursuit of a moving target than a shift of equilibrium. One consequence is the absence of correlation between the fertility decline and economic variables.

The mechanism stipulating that better longevity promotes investment in human capital is based on the idea that individual decision governs the relationship between longevity and education. I revisited this relationship from the perspective of optimal period school life expectancy. This expectancy is obtained by maximizing the utility of the population characterized by its age structure, mortality, and fertility by current age. It is necessary to use realistic mortality tables – contrary to what has been done – because the distribution of mortality by age plays an essential role, especially in child and juvenile ages. At maximum utility, current school life expectancy is a function of life expectancy and fertility. The application to French historical data from 1806 to today (taken from my book *Transformation of the French demographic landscape, 1806-1906*, 1997, Clarendon Press), shows that the age structure of the population has indeed changed the relationship between longevity and optimal schooling: in situations of high fertility and high mortality, lower mortality is associated with a reduction in optimal school attendance, because investing in education in age groups where only a small percentage will survive at productive ages is a financial loss; after a certain threshold during the demographic transition, when mortality and fertility have declined sufficiently, the relationship is reversed, and longevity then means an increase in optimal school attendance.[20]

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2.6 Learning hygiene and the beginning of the mortality transition: the major role of religions

Don Army Territory from 1867 to 1916 offers a unique opportunity to follow the variations in mortality between the different religious denominations (Orthodox, Old Believers, Catholics, Lutherans, Jews, Armenian-Gregorians, Lama Buddhists, and Muslims), in a context of difficult climatic conditions, urbanization and improvements in sanitation and médecine. Denominational groups differed in their lifestyles, housing segmentation, hygiene practices, and medical knowledge. The most educated and urbanized denominations

[20] Bonneuil, N. and Raouf Boucekkkine (2017) "Longevity, age-structure, and optimal schooling," *Journal of Economic Behavior and Organization* 136. 63-75.

had the lowest mortality rates. Religions acted as epidemiological barriers, supplementing the lack of doctors for the Orthodox, by enunciating hygiene rules on the faithful, by being associated with the precepts of life or through the doctrine of the fertility and the care of children.[21] To do this, we estimated systems of related regressions, one regression for each of the eight districts (*okrug*) and this for each denomination, because there is no reason for the relations to be the same from one district to another or from one religion to another (this is even what we are searching). The variables are time series (made stationary) of short-term fluctuations in temperature and rainfall, or economic like the price of rye. The dependent variable is the fluctuation of infant or child mortality, the difference from infant or child mortality between boys and girls, or life expectancy after 5 years per sex.

2.6.1 Religions, economics, politics, demographic transition in South Russia 1863-1916

In the case of the Don Army Territory, religious interdicts differed from one denomination to another, and within a same denomination, varied according to its degree of urbanization, over time, and from one district (*okrug*) to another. These contrasts reveal the segmentation of this society, the weight of the penalties related to agricultural work and military obligations, economic and urban differences and the degree of progress of the secularization, or the maintenance of tradition. They participate in the oppositions that will be revealed at the time of the revolutions of 1905 and 1917.[22] For the Jews, we showed the absence of regularity in the total number of marriages per Julian calendar month, then estimated the total number of marriages per Hebraic month thanks to a stochastic optimization in large dimension. It then appears that there is a strong seasonal regularity in the number of Jewish marriages.

[21] Bonneuil, N. and Fursa, E. (2017) Learning Hygiene: Mortality Patterns by Religion in the Don Army Territory (Southern Russia), 1867-1916, *Journal of Interdisciplinary History* XLVII: 3, 287-332.

[22] N. Bonneuil and E. Fursa (2013) "Secularisation and Confessional components of the seasonality of marriage in South Russia, 1867-1916," *Continuity and Change* 28(1), 51-88.

2.7 Seasonal patterns of mortality at the beginning of the mortality transition reveal the importance of waterborne and respiratory diseases.

Extracting monthly mortality from statistics of deaths by month and age allowed me to calculate the seasonal components of infant mortality rates. To do this, I had to disentangle the monthly statistics of deaths by age and of birth by articulating demographic equations and stochastic optimization.[23] For the Don Army Territory, for the period 1872-1915, these components reflect respiratory diseases in autumn and spring, dehydration and waterborne diseases in summer, and cold stress in winter. During the hottest months, they were lower in cities. Summer heat had a lethal effect, dampened by rainfall; infants were exposed to seasonal change; high winds increased mortality in late winter for infants 0-5 months and 6-11 months in cities. By the turn of the century, mortality had decreased, thanks to the gradual sanitation of water tanks and improved sanitation practices. We present relationships not only with monthly temperature and rainfall, but also with the monthly data of the wind force, which plays an important role in asthma, a disease that is twice as deadly for boys than for girls.

2.7.1 Seasonality of conception in traditional populations

Seasonality of conception in populations not using contraception has remained a *terra incognita*. First, I showed the influence of the seasonality of marriages on that of births, taking into account the stages in women's reproductive lives (risk of conception, pregnancy, seniority in breastfeeding) and the presence of cohorts of unequal numbers of people. I used the theorem of strong ergodicity to extend to all birth orders the dominance property of month 10 after marriage. I then showed that the monthly distribution of births from successive cohorts of unequal numbers has no longer month 10 necessarily as a mode, but that this distribution keeps the memory of the monthly distribution of marriage of each of the marriage cohorts. Finally, I reconstructed the monthly distribution of conception by age and period from monthly series of marriages and births. I used an optimization technique for this purpose under a Leslie-type recurrence incorporating a conception probability varying in age and time. The application to Armenian-Gregorian women of the Don

[23]N. Bonneuil and Elena Fursa (2017) "Learning Hygiene: Mortality Patterns by Religion in the Don Army Territory (Southern Russia), 1867-1916," *Journal of Interdisciplinary History* XLVII: 3, 287-332.

Army Territory (South Russia) from 1889 to 1912 reveals the high consistency between the reconstructed conception, the average age at marriage, and fertility. Only among these three measures, the reconstructed conception is free of structural effects by age and marriage calendar.[24]

2.7.2 The seasonality of age classes converges after fluctuations and mostly depends on those of births

The seasonal variations in age class sizes involve those of births and mortality across ages. They affect the censuses and, consequently, rates involving numbers by age. As their analytical expressions become inextricable, a simulation of ageing cohorts by months of age shows that mortality oscillations for human populations are not sufficient to prevent age classes from oscillating approximately like associated births, contrary to what previous literature suggests. The amplification converges after damping, and the level reached depends on the amplification of mortality oscillations relative to births between 0 and 6 months of age. The amplification of the oscillations of the age class sizes, in the course of ages, converges after damped oscillations. The damping rate depends mainly on the amplification of the mortality from [0, 6) months to births. The application to the 1896 South Russian data shows that the age class sizes vary during the year like the births of the associated cohorts and that the numbers counted at the census vary strongly according to the month of the census.[25]

2.8 The schooling revolution in 19th century France (controlled differential equations of sets)

In the social sciences, the concept of diffusion is used to describe the spread of a disease, the acquisition of a skill, or social changes resulting from innovations in customs, beliefs, tools, techniques, adopted by one people from another. Modelling diffusion is often based on chance that one member of the population is affected by another. However, in the natural and social sciences, processes can draw shapes that are not necessarily regular

[24] N. Bonneuil and Elena Fursa (2018) “Optimal Seasonality of Conception inferred from Marriage and Birth Time Series in Populations with no Contraception,” *Mathematical Methods in the Applied Sciences* 41(3), 1125-1135.

[25] N. Bonneuil and Elena Fursa (2022) “Seasonal fluctuations of age classes, with application to South Russia, 1896-1897”. *Mathematical Population Studies*.

sets.[26] As an exemplary case, schooling does not "spread" in the sense of a disease, or in the sense of a word of mouth custom. The process requires an investment in money, organization, personnel, it is modulated by the demographics of school-age children and by the filling of schools, an operation that also involves decisions and resource constraints. In addition, the difference between boys and girls betrays political and social orientations; competition between confessional and secular systems is regulated by law. The process borrows less from diffusion than from controlled spatial change. The interesting case is the schooling of France in the nineteenth century, a case well documented thanks to the critical work of statistics carried out by Jean-Noël Luc. Based on the concept of the derivative of a shape function introduced by Céa and Zolésio (1976) and that of graph derivative for correspondences forged by Aubin (1998) and of mutation of function, we can define the velocity of a set, and the kinds of controlled differential equations governing the variation of sets within the general framework of metric spaces (we leave linear calculus). In particular, the difference between the overall velocities in 1867 and what was observed in 1876 can be used to measure the effect of the 1870-71 war. The schooling of France thus generates dynamics of sets, here level sets, described by arrows of direction in each point of the space. Comparison of the observed overall velocities with those resulting from some investment policies (spatial convergence, maximization of the average enrolment rate, minimization of the girls-boys gap (Victor Duruy), investment proportional to the needs), makes it possible to locate the investments actually made.

2.9 The baby-boom/baby-bust in the 20th century: figure of time and viability

The history of fertility fluctuations in the 20th century has been thought in terms of cycles, particularly since the scenario proposed by the economist Richard Easterlin, who predicts the existence of cycles. Ronald Lee or Kenneth Wachter, in Berkeley, proposed a modelling in terms of an integro-differential equation that, near a certain equilibrium and with certain parameters, produces self-sustaining oscillations, which would ensure a perfect fertility prediction. From an empirical point of view, there is a long stagnation following a single "cycle" made of the post-war baby boom and baby bust. The mathematics of time has made great progress, which can be used to think about our historical and demographic

[26] N. Bonneuil (2014) "Morphological Transition of Schooling in 19th century France," *Journal of Mathematical Sociology* 38: 95-114.

time. The phase space analysis proposed by PoincarÃ© at the beginning of the 20th century, is from this point of view illuminating: it shows that the rhythms observed in the time series are structured around specific forms, revealing behavioral patterns, these identify themselves as attractors in the phase space. This analysis shows that the rise from a low to a high fertility regime occurred rapidly, and that the return was slower, with temporary stabilization halfway through. Everything happens as if, instead of linearly succeeding each other according to the formal chronology, the phenomena jumped over time from one regime to another. Instead of thinking in terms of time series, it is better to distinguish behavioral regimes, which correspond to attractors, which I have highlighted by using differential topology techniques (Takens' theorem, phase portrait, PoincarÃ© sections, first return functions). Where time series typologies were developed, I showed that a certain unity of time forms characterizes the history of European journeys.[27]

This work has paved the way for a re-reading of the past in terms of dynamic systems. Where Easterlin proposed a mechanistic relationship between economic well-being and reproduction, we know myopic agents in an uncertain environment. It is therefore necessary, somewhat contrary to certain ideas in vogue in physics according to which simple equations generate complexity, to highlight simple dynamics in the complexity of the social world. I proposed an explanatory diagram of the 20th century baby-boom/baby-bust based on constraints of inertia of economic variables, and viability of lifestyles. When many choices are possible, between consuming more, getting richer, or reproducing, there is a boundary of possibilities that must be avoided, because it leads to impoverishment: it is reached when one constantly increases one's consumption without getting rich fast enough and by maintaining a large family. This truism is deeper than it seems, because it allows a variety of different behaviors as long as a certain limit, called the viability boundary, is not crossed. The consequence of the model is that economic survival leads to the reduction of offspring. This model allows for discontinuous and unpredictable evolutions, as well as for a multiplicity of trajectories, which are geometrically different, but qualitatively similar, to well reflect the empirical data.

The World War brutally shook up consumption and reproduction. The distance from the sustainability boundary in the state space made it more interesting to increase one's offspring, while consumption was gradually improving, but slowly. The return to the

[27]Bonneuil (1989) "Conjuncture and structure in fertility behaviors," *Population English selection* 1, 135-158.

viability boundary at the end of the 1960s soon led to a new fertility decline.[28] This is an exemplary case where an economic event has created a long-lasting structure.

It is thus possible to put the mathematics of time at the service of our understanding of historical rhythms, and conversely, to find in history questions that intrigue mathematicians. I wanted to continue this inter-fertilization by exploring the more distant past.

2.10 Health over the life cycle

2.10.1 Infectious diseases: measles, AIDS, smallpox

In epidemiology always, in relation to the themes of the cycle of life, populations and the environment, I participated in the computation of the incubation period of AIDS,[29] to that of the determinants of measles in eastern Senegal, notePison G. and N. Bonneuil, (1988) "The Impact of Crowding on Measles Mortality. Evidence from Bandafassi Data (Senegal)," *Review of Infectious Diseases* 10(2), 468-470. highlighted the transfer of measles from boys aged 1 year old to girls and boys aged under 1 year old in Bamako, Mali,[30] and showed how dispersion and isolation allowed to some Amerindian tribes to survive epidemics in the Orinoco after contact with the Europeans.[31]

2.11 Management of queues to ventilators in the case of Covid-19

Triage protocols for intensive care units are based on priorities assigned to presents, but ignore patients about to arrive, so a priority newcomer may not find a ventilator available because it is occupied by a lower-priority patient who however was present at the moment of assignment. Conversely, waiting too long leads to losing elderly patients that could have been saved by ventilators. As age and sex are major determinants of mortality by Covid-19 and have the merit, in contrast to other priority criteria, of being immediately available to health professionals, the criterion is the minimization of the mean mortality rate weighted by age- and sex-specific life expectancies. The dynamic is a queuing process

[28] N. Bonneuil (1994) "Capital accumulation, inertia of consumption, and norms of reproduction," *Journal of Population Economics* 7, 49-62.

[29] Brouard, N. and N. Bonneuil (1992) "How reporting delay, duration of follow-up and number of cases affect the estimates of the incubation time of transfusion-associated AIDS," *Mathematical Population Studies* 3(3), 189-198.

[30] N. Bonneuil and P. Fargues (1989) "Predicting the vagaries of mortality: causes of deaths in Bamako 1974-1985," *Population Bulletin of the United Nations* 28, 58-94.

[31] Alexander M. Rodríguez and N. Bonneuil (1996) "Dispersiòn y asentamiento interfluvial: dos razones de sobrevivencia étnica en el Orinoco Medio del post-contacto," *Antropologica* 84, 43-72.

involving mortality and return home flows and competition between ages. The result is the determination of an optimal threshold age that can guide triage.[32]

2.11.1 Dependency at old age

Informal caregivers Provision of informal care is declining in South Korea. Applied to the 4-wave *Korean Longitudinal Study of Ageing*, I built a a Heckman selection model[33] to show that women with a dependent disengage more than men from caregiving; caregivers are young rather than old, and more frequently live in rural areas. They receive more financial transfers from non-cohabiting relatives and have fewer children than non-caregivers with a dependent adult. Both caregivers and non-caregivers are less likely than those without a dependent adult to give money to non-cohabiting relatives. Educational level does not influence the probability of having a dependent relative or, among people with a dependent, of being a caregiver. Among people with a dependent adult, the unemployed are more likely than wage earners not to provide care. Co-dependence is the only variable influencing the time spent in caregiving, which depends more on the dependent's demands for support than on the caregiver's capacity to provide it.

Transitions betwen dependency statuses With Younga Kim, we identify the determinants of transitions to dependency and the determinants of a possible return to non-dependency.[34]. Through multi-level logistic regressions, we characterize transitions from non-dependence to difficulties in daily living activities —instrumental[35] or not— in terms of stability, deterioration, death, or remission. Controlling for other effects, women are more likely to remain without difficult as men, dependent people more than women, and dependent people are likely to be excluded from the labour force. The unemployed, urban dwellers are more likely dependent. Subjective health helps predict health status two years later, as do grip power, cognition score, depression, heart and cerebro-vascular disease, and the number of difficulties in activities (but not instrumental). People with

[32]

Bonneuil, N. (2021) Optimal age-based management of the queue to ventilators during the Covid-19 crisis, *Journal of Mathematical Economics* 93, 102494.

[33]N. Bonneuil and Younga Kim, "Who (still) cares? Patterns of informal caregiving to dependent adults in South Korea 2006-2012," *Asian Population Studies* 16(1), 17-33.

[34]Bonneuil, N. and Kim, Y. (2019) Socio-economic and health determinants of transitions to adult dependency in South Korea, 2008-2014, *Public Health* 173, 130-137.

[35]personal grooming, housework, meal preparation, laundry, going out for short distances, traveling, shopping, money management, making phone calls, taking medications on time, dressing, washing your face, showering, eating, getting up, using the toilet, and controlling your bladder

difficulties only in instrumental activities and with no caregiver support survive longer, but are less likely to recover. Difficulties in daily living outweigh any other determinant for severe dependants. For them, mortality ceases to increase with age; it is no longer Gompertz. Cardiac and cerebro-vascular pathologies, smoking and alcohol consumption promote the transition toward dependance. Risk factors for dependency and the evolution of the status of dependency are evaluated, after consideration of other effects and selection bias.

2.11.2 Expression of genes over the life course

In epidemiology, I introduced the risk by age in the analysis of the genetic components of susceptibility to diabetes.[36] Until then, a constant age probability had been assigned to combinations of genes, while for some diseases such as diabetes or breast cancer, susceptibility varies with age. This considerable work has made it possible to more accurately assess the effect of some genes in the HLA group on diabetes, and has resulted in a software combining genetic segregation analysis and Cox model (in partnership with Antoine Clergé).

2.11.3 Genetic components of aging

I also proposed[37] the construction of a law of aging at the DNA level in the human brain on the basis of 12,625 genes of the frontal neo-cortex of 30 post-mortem individuals aged between 26 and 106 years. I used the analysis of survival to calculate the times of transition from activation to deactivation or conversely, with interval censoring, multi-level dependency, and unobserved heterogeneity. A Monte Carlo simulation allowed me to build an aging table by physiological function (amino acid modification, Ca2+ homeostasis, DNA repair, hormonal, CdK5, inflammation, MAP kinase cascades,...). I characterized the expression of genes with age, tested the differences in aging between groups of genes, deducted "laws of aging" at the genetic level. The intensities of transition and age-specific survival functions are calculated by physiological function and by sex from the transcripts. I thus built a continuous vision of aging in the human brain, differentiated physiological

[36] N. Bonneuil, A. Clerget, and F. Clerget-Darpoux (1997) "Variable Age of Onset in Insulin-Dependent Diabetes Mellitus by the Marker-Association-Segregation-χ^2 Method," *American Journal of Human Genetics*, July 61, 223-227.

[37] N. Bonneuil (2007) "Ageing Laws for the human frontal cortex," *Annals of Human Biology* 34-4, 484-492.

functions with respect to age. The number of activated genes decreases with age for all functions, except those associated with hypertensive tension and myelination/lipid metabolism.

2.11.4 Child Development

Using differential topology techniques, I have clarified the learning of the young child in his or her psychomotor development. For this, having a follow-up at different ages, against the usage based on statistical descriptions or Fourier decompositions, I proposed to preserve the non-linearity of the markers by constructing the first return functions, a technique borrowed from the theory of dynamic systems. With Blandine Bril, we have shown that the age-related development is manifested by a drift in dynamic structures, each determined at a moment of the child development. Behavior modification of the young child during his or her lifetime is a remarkable case of dynamics over age of dynamic structures over time.[38]

2.12 Emotions and populations

2.12.1 The mathematics of emotions

Rooted in survival reflexes and the struggle for reproduction, emotions are linked to the life cycle and demographic events (search for a spouse, marriage, pregnancy, illness, marital life, bereavement, . . .). Emotions make their mark in the time of life and death. Their very definitions make them dynamic systems with potentially varied and contingent consequences under viability constraints, meeting the principles of maintaining or acquiring desired properties.[39] I classified most emotions by their membership in sets of viability, which in turn include the quantifiers 'it exists' ('$\exists$') and 'for everything' ('$\forall$'). Writing emotions in this way makes use of most concepts of viability theory, because both emotions and viability sets deal with survival and change. Emotional regulation reflects mathematical commands, which can be implemented in a variety of ways, optimally or not, and can be improved through learning. The study of emotions staggering demographic events in the life cycle show the sequence of viability kernels and capture-viability basins succeeding

[38] Bonneuil, N. and Bril, B. (2012) "The Dynamics of walking acquisition," *Infant Behavior and Development* 35, 380-392.

[39] Bonneuil, N. (2015) "Emotions as Dynamic Systems in Viability Sets," *Mathematical and Computer Modelling of Dynamical Systems* 21(5), 460-479.

each other.

2.12.2 Courtly love: change of emotional regime in relations between the sexes

I interpret the arrival of courtly love in the twelfth century as an innovation from the Gregorian emotional regime, itself obtained by coercion from the Carolingian emotional regime.[40] I show that the redistribution of power in the intimate sphere has transformed the emotional equilibria governing intimate relationships. I show that the courteous regime possesses the Nash property, unlike Christian marriage, whose virtues *fides, proles, sacramentum* according to Augustine are principles of maintenance rather than maximization. I propose that the Gregorian order, requiring duty of reproduction with moral restrictions, is a saddle point in the mathematical sense. It is made stable by the control, namely the surveillance of priests and a part of society docile to religion. I show that the Church has increased transaction costs, that it has prioritized the moral value attached to desires, disqualifying greed or carnal desire, while placing the impulse towards God above those of the sensitive. Courtly love appropriates a moral value by increasing the cost. I further elaborate on the dynamics of the emotions involved in Gregorian marriage or courtly love, on the substitution of "at least one desire" towards the woman in the Carolingian regime for "all desires" towards God (Christian love) or towards the lady (courteous love), on the implementation by the Church of counter-emotions,... I also show that the Gregorian emotional regime shows its robustness by adapting to the deviance of troubadours, unlike courtly love, which degenerates into satire and farce set in the fabliaux, not without having opened the way to romantic love and outlined the erosion of God's exclusivity. I show similarities between courteous love as an innovation in intimate and social relationships and innovation in biological systems. Finally, I show that the time of the West of the twelfth century, as far as romantic and sexual relations are concerned, is structured according to the concept of capture (through the mathematical sets of "capture basins" and "absorption basins"), in contrast with the romantic relations in Bengal of the twelfth century, based on maintenance by repetition and sexual fusion in a certain set defining devotion, on the absence of dualism between here-and-now and objective-later (which defines a specific dynamic in terms of viability sets), and in contrast also with Japan at the

[40] N. Bonneuil, (2016) "Arrival of courtly love: moving in the emotional space," *History and Theory* 55(2), 253-269.

Heian epoch (794-1185), based precisely on the renunciation of the possibility of acquiring what is definitely lost (which defines a specific dynamic in terms of viability sets).

2.13 Viability in population economics

2.13.1 Women's labor market, labor economics

Personnel in a small firm 1946-1986 In personnel demographics, I analyzed a small family construction firm in the city of Nogent-sur-Marne from 1946 to 1986 (using a multi-process multi-level multi-spell model).[41]

This is probably the first time study of a small firm over such a long period and with such precision, in a very important sector and in a the context of migration and foreign labor market. The dynamics of the labor market are proving to be very different from that of a large company. The period was filled with economic ups and downs. Although family ties facilitate access to employment, kin to the owners of the firm were not paid more than non-kin. Promotions came more from the economic context, and the change in wage levels more from qualification than from kinship, seniority, or national origin. On the contrary to what has been observed for firms of large size, the internal labor market had remained informal; the need to respond to the market had fostered competence on seniority and kinship.

Age-Competence-Shift in a firm with high mobility The "Age-Competence-Shift" measures variations in the total wage bill from one year to the next due to changes in the structure of the corresponding workforce. For the population of an enterprise with high labour turnover: frequent changes in function, levels of qualification, seniority, type of activity, where arrivals and departures occur daily, the calculation of key elements, such as the Age-Competence-Shift, affecting the total wage cost, becomes difficult. For this reason, I have designed and written a software to computerize a solution.[42] The Company Générale Maritime acquired it for its flight crew who is highly mobile, sometimes daily. I have resolved this problem that was unaffordable by the method used in the context of slow careers.

[41] N. Bonneuil and Manuela Martini (2015) "Career Advancement in a Family-owned French Construction Firm under changing labor legislation and market demand, 1946-1985," *Family Business Review*, 28(1), 41-59.

[42] N. Bonneuil (1989) "Démographie du personnel : effet de structure sur l'évolution de la masse salariale dans une entreprise à forte mobilité," *Population* 6, 1101-1120.

The female labor market With my student Michael Grimm, we analyzed the 1990 fertility survey to accurately describe the determinants of transitions in the female labor market.[43] We have shown that the most educated women are quick to leave their employments, but also the fastest to take one, especially when they have work experience. Marriage, with or without maternity, is associated with a reorganization of time between home and work, a more likely exit from the labor market and a less likely return to work. Mothers from large families leave less often their employments, because they are less likely to return that have more children. Involuntary departures are more frequent in period of recession.

Precariousness of the female labor market Theories of precarious employment based on distinctions of quality and stability of employment have been made raised the issue of transitions, related to sex and age, from long-term employment of poor quality to stable employment of good quality. With Younga Kim, we characterized the transitions on the labor market over the ages by Markov matrices corresponding to the 8860 possible combinations of explanatory variables. These matrices show the importance of distinguishing the conditions on the labor market, which are shaped by the forces of the moment, from the conditions inherited from the past. The traditional position of women in the labor market has resulted in precariousness linked to age and sex, while current conditions generate a trend towards less precariousness in employment. Transition matrices between types of precarious employment, taking into account the duration spent in this employment by age group, the date and explanatory variables, provide the distributions of asymptotic prevalence, which reflects the conditions on of the moment on the labor market. The forces of the moment favor the predominance of stable employment of good quality, while the prevalence observed at a given date is characterized by the polarization of the labor market between stable, employments of good quality and unstable employments of poor quality. The asymptotic prevalence reveals a steady increase in the number of stable but poor quality employments. Older women are observed especially in unstable employments of poor quality, but labor market conditions tend to reduce this age divide over time. The conditions of the moment reduce the proportions of older women in stable

[43] M. Grimm and N. Bonneuil (2001) "Labor market participation of French women over the life-cycle, 1935-1990," *European Journal of Population* 17-3, 235-260.

low-quality employments and in unstable employments of good quality.[44]

Health and career path In the two-stage survey 2006 and 2010, I showed that income inequalities, associated with income mobility, are lower than those observed in cross-sectional sections in 2006 and 2010. Income mobility between 2006 and 2010 implies a more egalitarian distribution than that observed on each of these two dates. The rising gradient in inequalities observed over the life cycle disappears for the distribution of inequalities associated with conditions of the moment. Poor health always promotes inequality, especially since health deteriorates faster for low-income people. Poor health exacerbates inequalities, especially as income distribution is more unequal. By building a stylized but precise model, I corroborated the econometric analysis and calculation of empirical Markov matrices. In this context, I gave the analytical expressions.[45]

2.13.2 Economic viability of the life cycle

I have taken up the life cycle issue from a critical perspective of Friedman's, Modigliani's, or Carroll's economic theory of the life cycle, based on the 1997 survey on patrimony and with the modern tools of viability theory, where I propose an alternative to optimizing intertemporal utility. [46] Having children, guaranteeing a certain standard of living, and retiring with a certain amount of capital leaves room for many trajectories, where couples are divided between caution for their old age and for their children, and impatience to consume, under the threat of unemployment or poor returns on their savings. The heterogeneity of agents is reflected both by the entire state space where each state corresponds to a different situation, and by the set of achievable states, which reflects the uncertainty inherent in both decision-making and external shocks. The determination of all the states from which this program in consumption, reproduction, and saving, against the clock of age, can be carried out, in turn identifies the timely decisions of when to have children, when and how much to consume and save. The discontinuous aspect of consumption after birth is fully taken into account through continuous-discrete differential inclusions. I compare theory with the data of the patrimony survey, then to international data to show that

[44]N. Bonneuil and Younga Kim (2017) "Precarious employment among South Korean women: Is inequality changing with time?," *Economic and Labour Relations Review* 28(1), 20-40.

[45]N. Bonneuil, (2018) "Health Component of Inequalities associated with Income Mobility over the Life Cycle," *Social Indicators Research*.

[46]N. Bonneuil and P. Saint-Pierre, "Beyond Beyond Optimality: Managing Children, Assets, and Consumption over the Life Cycle," *Journal of Mathematical Economics* 44(3-4), 227-241.

lower fertility is associated with smaller sets of decisions made in time (instead of looking at the determinants of fertility as usual, I propose a reversal of perspective: for a given parity, I look at all the paths that lead to it, and how the set of these paths varies with the explanatory variables).

2.13.3 Anticipated survival and subjective quality of life in the trade-off between consumption and savings after age 45

The trade-off between consumption and savings takes place in a context of an ageing society where solidarity family is deteriorating. Maximizing utility allows us to calculate the preference for savings.[47] Arbitrage involves subjective satisfaction with quality of life, anticipated survival and the profiles of consumption. Endogenous econometric equations based on the *Korean Longitudinal Study on Aging, 2006-2014* (10 205 adults aged 45 and over) show that the preference for savings is determined only by and through these endogenous variables, with no other effects direct socio-economic benefits. The people who spend the most in education are those with the greatest preference for savings. Socio-economic variables influence the preference for savings in accordance with theory economic life cycle, but through the structured filter endogenous subjective variables and consumption profiles.

2.13.4 Cognitive bias to anticipate mortality affects subjective quality of life and consumption style

How does attitude to death influence consumption level and profiles and conversely?[48] The database is a representative follow-up of Korean respondents from 2006 to 2016. The question asked in the survey is about anticipated survival (respondents express their chances that they feel they will be alive in 15 years as a percentage). To characterize the attitude toward death as pessimistic, realistic, or optimistic, one must compare the anticipated probability of dying with the expected probability of dying (given by the life table of the moment). Indeed, respondents are aware that their risk of dying increases with age, but this observation does not take into account the fact that people consider that

[47] Bonneuil, N. and Kim, Y. (2021) Arbitrage between consumption nd saving for bequest: the role of subjective expected survival and satisfaction with the quality of life, South Korea 2008-2014, *Macroeconomic Dynamics* 25(4), 998-1019.

[48] Bonneuil, N. and Younga Kim (2022) Cognitive bias in anticipating mortality risk affects the subjective quality of life and consumption-related lifestyle. *Journal of Human Behavior in the Social Environment.*

they are living longer or shorter than expected (according to the current year's life table). The relevant measure, then, is the difference between the anticipated mortality underlying anticipated survival and the expected mortality based on the current national period mortality table by sex. It is net of age and age horizon effects. It is important because the underlying anticipated mortality conditions satisfaction with the quality of life and associated consumption patterns, and therefore, on a macro scale, the supply of economic goods to people. We have combined satisfaction with quality of life with consumption levels and patterns, the gap between anticipated and expected mortality, and wealth, all of which are endogenous variables. Exogenous variables include sex, age, education, children, place of residence and employment. We find that satisfaction decreases with age; women are less satisfied and anticipate a higher mortality than men. People anticipate a probability of dying that increases only slightly with age, whereas mortality actually increases exponentially. This cognitive bias, due to the fact that individuals experience linear progressions but have difficulty projecting themselves into the future, leads them to overestimate their satisfaction in terms of quality of life. This creates a protective effect on well-being, at the expense of a fair economic view of the life cycle.

2.13.5 Pensions

Pays-as-you-go Pension payments are compromised by the retirement of the large age classes born during the baby boom, which are followed by the currently depleted active classes. The numerical imbalance between contributors and pensioners is accentuated by the steady increase in life expectancy at birth. The common procedure is to simulate a priori scenarios, exploring a (necessarily) small set of plausible dates and values on changes in important variables, eliminating those that bankrupt the distribution system, and crossing fingers to make one of the tried scenarios work. However, the space for possible changes has the number of state variables for dimension: for example five if we consider the duration and amount of contributions, the rate of return, the unemployment rate, and the ratio of the number of pensioners to the number of contributors. This ratio varies over time with mortality, migration, and the transition to activity of the youngest age classes. In a space already of dimension two, it is very long to try all the scenarios, by which variables are modified, one by one. The alternative procedure I developed[49] is to determine all states

[49] J.-P. Aubin, N. Bonneuil, F. Maurin, and P. Saint-Pierre (2001) "Viability of Pay-As-You-Go Systems," *Journal of Evolutionary Economics* 11, 555-571.

from which there is a solution to maintain a sufficient income for pensioners and for active people for a sufficient period of time, while respecting a certain equity between generations, and by not underestimating the resistance of the actors to social change. Actions to achieve the maintenance of the system within the specified constraints include the extension of the contribution period and the increase in contributions, for given unemployment and interest rates. One of my results is to indicate what are the viable policy actions, when to implement them, and to what extent. The difference with traditional optimization procedures is the taking into account of constraints and broadening the response to all viable policy actions.

By capitalization With regard to funded retirement, the financial equilibrium of pension funds is based on the correct calculation of retirement pensions, taking into account future payments and discount rates. Short-term errors in pension commitments to be paid, inappropriate investments in financial markets, or a mismatch with pay-as-you-go payments have long-term negative consequences and can lead the pension fund to bankruptcy (as in fact, this is often the case in France). I have established[50] a differential equation governing the assets in order to update the respective weights associated with the error made on the interest rate, the error made on undue (for commercial reasons) allowance of points, and the error made on mortality forecasting. A brief follow-up is sufficient to estimate these three errors. I determine a minimum interest rate to earn on the financial markets to offset the supplement of awarded points.

2.13.6 Generational equity

Likewise, preserving intergenerational equity means regulating intergenerational transfers so that no generation spends more than it receives: I took up this problem again by avoiding optimizing an intergenerational utility established on an infinite future, but by reformulating the question in terms of viability. This results in a rate of return optimal of "human capital": for low values (between 4 and 11% in the French case), it is all the easier to ensure a positive net present value for each generation that this return of human capital is growing; but a rate of return that is too high (above 11%) creates a gap between pensioners and active people, and leads trajectories to violate the constraint of equity:

[50] N. Bonneuil (2013) "Early warning to insolvency in the Pension Fund: the French case," *Risks* 1, 1-13.

viable states are becoming rare, the viability kernel is decreasing.[51]

2.13.7 The dynamics of vintages

Vintage models in economics deal with age structures in the ageing of physical or human capital. They raise the additional difficulty of differential equations with delays.[52] I calculated the viability kernels (the sets from which there is at least one (controlled) trajectory preserving the system) and estimate the influence of their determinants (consumption threshold, technological rate, investment). In particular, I have shown that the inverse relationship observed by Boucekkine et al. (1998) between the duration of obsolescence and the rate of technological progress, valid on the optimal paths, is also valid in the viability kernels. For that, there is no need to solve Hamilton-Jacobi-Bellman. In addition, I take into account the heterogeneity of agents, represented by the state space, and the heterogeneity of eligible investments. I also broaden the framework of the hypothesis of rational agents (who need to be viable to survive economically, whether they are optimizers or not).

2.13.8 The origin of economic preferences

I have addressed the origin of preferences,[53] from the work of Robson and Samuelson who showed how long-term preferences result from dynamics governed by Leslie's matrices. However, in the a changing environment, these matrices change, and their dominant eigenvalues no longer give surviving long-term preferences. What are the preferences that survive in the long term? How to account for the diversity of preferences in time and place? An example is the timing and intensity of fertility. I also show how to replace surfaces of indifference (which can be described by a strictly utility increasing and quasi-concave function) by the sets of viable regulations (ordered according to strict inclusion and corresponding to closed graph dynamics).

[51] N. Bonneuil and Romina Boarini (2004) "Preserving Transfer Benefit for Present and Future Generations," *Mathematical Population Studies* (G. Feichtinger and V. Veliov editors) 11(3-4), 181-204.

[52] N. Bonneuil (2010) "Viability and Optimality in Vintage Models," in N. Hritnonenko, Y. Yatsenko, and R. Boucekkine (eds), *Optimal Control of Age-structured Populations in Economy, Demography, and the Environment*, New York: Taylor and Francis.

[53] N. Bonneuil (2010) "Diversity of Preferences in an Unpredictable Environment," *Journal of Mathematical Economics* 46, 965-976.

2.13.9 Endogenous growth

I used Ramsey's model of economic growth in the prospect of viability. Increased by a criterion of minimum consumption and a sustainability criterion, it becomes a viability problem, whose solutions include viable optimal paths, optimal paths and non-optimal viable paths. The sacrifices in terms of current consumption required for the sustainability according to the Brundtland Commission show that rich countries must take most of the burden. Econometric analysis of the volumes of viability kernels emphasizes the role of technological progress in ensuring sustainability according to Brundtland. The preference parameters such that the pure time preference rate is statistically not significant.[54]

2.13.10 The commons

Populations and energy transition I examine the relationship between population and energy transition.[55] The time horizon plays a role comparable to the maximum fertility age and the times at which decisions are made are controls, as are the times at which children are born in the life cycle theory. Change management is carried out by solving a six-dimensional viable optimum problem. The initial conditions of the population, its fossil fuel consumption, the initial level of pollution, and available time determine the timing of technological change over the entire period. Finding the best time for regime change leading to gradual change is an original problem. With my co-author, we give an elegant solution thanks to the theory of viability, my theorem on the viable maximum, and the algorithm I developed.

The mathematical viability of the commons Population growth changes the optimal equilibrium between the stationary population and its expected resources, producing a balance curve instead of a single point. This curve is characterized by a variable number of people, a variable quantity of resources, a variable harvest per head, and variable birth rates.[56] I give an analytical expression of Nash equilibria for two populations sharing a common resource capable of growth itself. I develop an alternative procedure, which avoids solving differential equations and intrinsically includes state constraints. This leads to the

[54] N. Bonneuil and Raouf Boucekkkine (2014) "Viable Ramsey economies," *Canadian Journal of Economics*, 47(2), 422-441.

[55] N. Bonneuil and Raouf Boucekkkine (2016) "Optimal transition to renewable energy with threshold of irreversible pollution," *European Journal of Operational Research*, 248, 257-262.

[56] N. Bonneuil (2018) "Population Growth and Nash Equilibria under Viability Constraints in the Commons," *Journal of Optimization Theory and Applications* 176, 478-491.

construction of the capture-viability kernel of an auxiliary system. For two populations, I demonstrate that the set of all Nash equilibria under state constraints is the intersection of the edges of the two capture-viability kernels. The two methods, Pontryagin (in this rare calculable case) and viability, give very consistent results, which confirms the accuracy of my two theorems, my algorithm, and its application. Viability provides viable Nash balances (unlike Hamilton-Jacobi-Bellman or Pontryagin); its scope is large (only Lipschitz and convexity of images are required); and it is more flexible to use because it avoids solving differential equations for each initial condition.

Nash equilibria in the pollution of the commons I deal with the problem of the commons in a non-linear framework and under constraints of viability. The protagonists can change dynamics during the differential game. I calculate all viable Nash equilibria using an innovative method, based on intersections of varieties in large spaces, and using my 2006 viability theorem published in *J. Math. Analysis and Applications* and on the 2012 maximum sustainable maximum published in *Optimization*. At Nash viable equilibrium, populations are interdependent: the different levels of productivity lead the most productive to hasten its energy transition and the least productive to delay it. In the absence of cooperation, the efforts of some demotivate or delay others, with consequences for the well-being of populations.[57]

Nuptiality to regulate the commons? The case of the Don Cossacks (south Russia), 1867-1916 Sustainability in the commons has been associated with the optimal net present value controlled by the harvest rate under stationary population. Population growth however disrupts this scheme. In traditional societies, fertility was regulated by age at marriage. In times of population growth and limited resources, economic sustainability then requires that age of marriage should be raised. In the case study of the Don Cossacks, 1863-1916, with Elena Fursa I showed[58] that early marriage, which was an important marker of social cohesion, was too slow to increase when mortality declined, fueling a population growth that threatened the agrarian economy: age at marriage then appears to be essential to the theory of the commons in traditional societies. For this purpose, I introduced age at marriage as an additional control in an operational research model

[57] N. Bonneuil, and Raouf Boucekkine (2017) "Viable Nash Equilibria for Common Pollution," *Pure and Applied Functional Analysis* 2(3), 427-440.

[58] Bonneuil, N. and Elena Fursa (2021), Nuptiality to Regulate the Commons? The case of the Don Cossacks (south Russia), 1867-1916, *Oxford Economic Papers*, 73(2), 698-719.

anchored in historical data, under Lotka-McKendrick dynamics and resource harvesting. We also supported the analysis with archival data on land prices by quality level, yields, and climatic variations.

2.14 Social networks

Regulations can also be connection matrices. The success of network analysis in social science is based on the observation that certain social events that are poorly explained on the basis of socio-economic considerations become more understandable when linked to networks of relationships. The now classic procedure is to estimate matrices of ties. Some authors have suggested linking the existence of ties to socio-economic variables, and to discern dominance or substitution relationships in the construction of networks. This is like considering social networks as variables in network spaces. I proposed a different view[59] of dynamic networks, showing that these are used less as state variables than as control variables in controlled systems. These are therefore controls in spaces of large dimension. In Sampson's famous study of a community of monks, I have shown that the state variable is the willingness to stay in the monastery. When a crisis occurs, the network of friendships or of enmities governs the order of departure of the monks. The social crisis is a crisis of viability of the system. Thanks to viability inequalities, I have reconstructed the underlying processes that lead to the observation of the event.

I took over a second famous article showing that the political control of the Medici in 15th century Florence ("everyone knew that the Medici wanted, as bankers, to make money; as families, to increase prestige; as neighborhood patrons, to amass power") resulted from a network of business and marriage. I have shown that the network actually used satisfies the conditions for the viability of the domination of the Medici. These conditions determine all networks that would have made the same success possible: the networks are then strategies in the struggle for power, and the variable of the state is political power. It is then a question of using viable networks, through which the time trajectory will bring political control to the Medici: these networks satisfy the conditions of viability. Padgett and Ansell (1993) showed that the Medici drew their power from the eminently central position that they occupied in the network: I showed that the centrality of the Medici is only one way to obtain viable matrices in the sense of their political domination.

[59]N. Bonneuil (2000) "Viability in Dynamic Social Networks" *Journal of Mathematical Sociology* 24(3), 175-182.

At any given moment, a whole set of networks is possible for the Medici, not only the one retained by the chronicle: the Medici could establish new links, break new ones, a whole set of changes was possible, but only those that are viable are beneficial to the Medici. Their cleverness was in their ability to establish and to select the links that make the connection matrix viable.

3 The maintenance of genetic polymorphism

3.1 Genetic recombination and natural selection

In population genetics, a fundamental issue is the maintenance of polymorphism in time-varying environments. Density-dependent selection or selection by stochastic variation have been proposed as participants in the selective factors of molecular variation. In many natural biological populations, the selection coefficients are not constant but fluctuate from generation to generation due to genetic transmission and changes in the environment.

In an unpredictable environment, there is no longer any question of looking for maximum fitness, but the concept of viability kernel accurately reflects the issue of maintaining the polymorphism. It allows taking into account transient changes, which are the rule in population biology, and achieving results without imposing laws on variations of fitness: the viability kernel therefore plays a central role in the understanding of Darwinian selection, because the trajectories leaving this set enter a crisis of scarcity, while those still in the kernel retain a chance of keeping the system in a sufficiently polymorphic state. Systems do not “select” fitness values. The viability kernel simply reveals that, if the alleles reach outside the kernel, then polymorphism is doomed to impoverishment.[60]

3.2 Regaining bio-diversity

The maintenance of genetic variation raises the subsequent question of the advent or return of a rare allele, an important springboard for genetic conservation. The concept of a minimum number of generations in impoverished polymorphism provides an answer. It allows to decide on the existence of a road to polymorphism, and to determine which

[60] N. Bonneuil and P. Saint-Pierre (2000) “Protected polymorphism in the two-locus haploid model with unpredictable fitnesses,” *Journal of Mathematical Biology* 40(3), 251-277.

fecundities associated with allele combinations should be selected in each generation.[61] The plotting of the contours of the minimum number of generations outside polymorphism reveals the importance of heterozygous fertilization.

3.3 Migration and selection

I reformulated the maintenance of polymorphism under selection and migration varying over time as a viability issue.[62] I have identified the viability kernel for migration and soft selection, from the case of two demes and two alleles up to the case of four demes and four alleles. I showed the determinants of the maintenance of polymorphism in this situation of unpredictable fitness and migration. The viability kernel combines the tension between opening and closing to migration to the set of fitness values of all alleles in each deme. These determinants are set-valued: along the trajectory on which polymorphism is maintained, fitness values and rates of migration are not fixed by a rule (for example constant, endogenous, frequency dependent, cyclical, random) but simply belong to the range of admissible values. Some values will be taken repeatedly: the repetition of low enough or high enough fitness values and migration rates has no fixed rule of occurrence; it depends on where the system travels in the viability kernel. I therefore suggest that the search for regular patterns in empirical systems could be supplemented by the search for simple repetitions. Inside the set defining polymorphism, the remaining trajectories are myriad, and the sequence of viable controls depends on the taken trajectory. This contributes to the uncertainty of the dynamics, without resorting to any probability law or any endogenous mechanism or sensitivity to initial conditions.

3.4 Genetic diversity and viability under stochastic dynamics

In population genetics, in the Moran model of drift and election of a mutant allele with population growth, instead of examining the consequences of predefined selection and population growth, the coexistence of the wild allele and the mutant allele becomes the maximization of the sojourn time in a given set.[63] I proposed to extend the concept of

[61] N. Bonneuil and P. Saint-Pierre (2002) "Minimal Number of Generations out of Polymorphism in the One-Locus Two-Allele Model with Unpredictable Fertilities,", *Journal of Mathematical Biology* 44(6), 503-522.

[62] N. Bonneuil (2012) "Multiallelic Polymorphism Maintained by Unpredictable Migration and Selection," *Journal of Theoretical Biology* 293, 189-196.

[63] N. Bonneuil (2022) Optimal control of genetic diversity in the Moran model with population growth, *J. of Biological Systems*

viability: A state x_0 in a given set K at the horizon T is viable if the maximum sojourn time in K starting from x_0 is T. In the stochastic setting, I proposed to replace this concept by the maximum sojourn time in K starting from x_0. The process is driven by the additional controlled mortality of the mutant and by population growth. This allows for assigning fitness values as feedbacks of the constraints, thus orienting a conservation policy or a strategy towards a wishful proportion of mutants, or, alternatively, giving the optimal conditions that have allowed coexistence.

3.5 The economic value of bio-diversity

Conciliating genetic conservation with economic profitability requires systems theory dynamics under viability constraints.[64]. With Raouf Boucekkine, I proposed that the economic value of the distribution of alleles is viable if genetic diversity is preserved along the associated time path. The largest set of initial distributions from which there is at least one trajectory preserving genetic diversity —understood as a set determined in the state space–is the capture-viability kernel. It provides a decision-making rule on how to regulate the resistant and the susceptible strains, depending on the uncertain and time-varying presence of the parasite. It is necessary to know how to prevent the fixation of the susceptible allele in the absence of the parasite to maintain genetic diversity when the parasite reappears. The variation in the capture-viability kernel reveals the influence of determinants (prevalence of disease, cost of virulence, cost of inability to infect the resistant host, cost of adaptability when a resistance reaction is triggered, cost of adaptability for resistant hosts which do not encounter pathogens) within this general time-varying framework. A higher value is obtained by starting from a large genetic distance when the infectious parasite is sufficiently abundant ; in its absence, the value is greater when starting from a low prevalence of the resistant allele, which corresponds to a small genetic distance. Among the viable trajectories, there is one that provides the largest inter-temporal benefit. I determine this one by calculating a capture-viability kernel for an augmented dynamic, thanks to my theorem in *Optimization* 2012.

[64] Bonneuil and R. Boucekkine (accepted) “Genetic diversity and its value: conservation genetics meets economics,” *Conservation Genetics Resources*.

3.6 Dynamic games of populations

The other historically fundamental model of population dynamics is that of interactions between biological species. In dynamic game theory, each player can modify his strategies at all times, taking into account the actions of the other players. His goal is first to stay in the game, and if possible to be in its *victory domain*, the set of states of the game where there is a winning solution that takes into account the strategies adopted by the other players.

The prey-predator model can be considered as such a dynamic game. Coming from bio-mathematics, this model has numerous applications in economy (see Friedman in *Econometrica*, 1991) and in demography (Micheli, 1994; von Tunzelman). The mathematics of this model is therefore of interest to the social sciences. Initially considered constant, interactions between prey and predator actually are subject to constant fluctuations due to stress, the changing environment, genetic mutations, or the phenotypic heterogeneity of populations. We could thus imagine these interactions as functions "endogenizing" densities, or as stochastic functions,... As we actually ignore the variation of these interactions, the only thing that we can figure out is that they vary within sets, which cover our uncertainties or our ignorance. The challenge for the prey is to survive, while the predator or super-predator must be careful with the prey to ensure its long-term survival. The looters must spare the peasants if they want to be able to ransom them again later. These considerations of constraints on states and strategies are sufficient to delimit the largest viability domains for each of the protagonists, as well as their coexistence domain. I also determine which strategies ensure the viability of one or the other protagonist (sometimes there is only one pair, sometimes a larger set –the system regulation is set-vales).[65,66]

[65]N. Bonneuil and Katharina Müllers, 1997, "Viable Populations in a Prey-Predator System," *Journal of Mathematical Biology* 35, 261-293.

[66]N. Bonneuil and P. Saint-Pierre, 2005, "Population Viability in Three Trophic-level Food Chains," *Applied Mathematics and Computation* 169/2, 1086-1105.

4 The mathematics of the time of populations: theorems, algorithms, theory of history

4.1 Existence and uniqueness of Lotka-McKendrick solutions

Webb (1981) demonstrated the existence and uniqueness of the solutions of the system consisting of McKendrick's differential equation and Lotka's integral equation, at the heart of the mathematics of populations,[67] in the framework of integrable solutions in the sense of L_1. With Jean-Pierre Aubin and Franck Maurin, I extended the theorem of existence and uniqueness of solutions to the case of functions with closed graph.[68] In particular, the solutions now appear as running through the sets of attainable states from the initial pyramid and the births. The set of these sets of attainable states is *the invariance envelope* of all constraints, whose existence and uniqueness derive from purely topological properties. The compliance of the system with constraints over time comes from the very construction of the invariance envelope. The demonstration is about twenty pages long, instead of a full book in the case of L_1-integrable functions, and the forces of mortality, migration and fertility are very general.

4.2 Viability multipliers generate models from constraints

Most population models are based on conjectures consisting of postulating interactions (logistical, endogenous, ...). The non-linearities that have made the fortune of population models produce interesting trajectories, but are neither necessary nor sufficient for maintaining populations. I have shown how the viability conditions are inherent in maintaining populations.[69] They extend to the Lagrange multipliers method to non-smooth sets of state constraints. They are used to select the models whose solutions meet state constraints. In particular, I have shown how the historical experiences of Gause, Luckinbill, and Pimentel, who start from prey-predator systems dedicated to extinction and modify them in such a way as to make them persist, amounted to building viability multipliers (respectively by migration, alteration of the environment, or genetic adaptations).

[67] On which I committed a textbook: N. Bonneuil (1997) *Introduction à la modelisation demographique*, Paris, Armand Colin.

[68] J.-P. Aubin, N. Bonneuil, and Franck Maurin (2000) "Non-linear Structured Population Dynamics with Co-Variates" *Mathematical Population Studies* 9(1), 1-31.

[69] N. Bonneuil (2003) "Making Ecosystem Models Viable," *Bulletin of Mathematical Biology* 65, 1081-1094.

4.3 My viability algorithm: the challenge of the large dimension

The algorithm used to calculate the viability kernel was limited to three dimension, because each point of a grid covering the set of constraints must be iteratively tested and its viable or non-viable status stored. I proposed a new algorithm to process large state dimensions.[70] I no longer store a grid, but define a distance associated with a initial condition and a solution governed by a control path. By stochastic optimization on the sequence of controls, I minimize this distance, which allows me to identify if the initial state is viable or not. The search for the initial state is also carried out by stochastic optimization, so that the procedure consists of a double stochastic optimization: one with examining the initial state, to decide whether it is viable or not, and the other where the initial state varies. A semi-permeability property (a solution can leave the set of state constraints but not enter it) allows collecting several other viable states once a viable state is obtained on the edge of the set of state constraints. The volume of the viability kernel and its confidence interval are calculated by drawing states randomly and by identifying their status of viability. This makes it possible to characterize the volumes, which reflect the room of manoeuvre of the actors, with respect to the parameters.

4.4 The viable optimum

I then examined[71] the question of maximizing while remaining viable a function of the solution or an integral part of this function (Mayer problem with viability). I have established a theorem to calculate all the maximums as the boundary of a certain set (the domain of capture-viability of a certain dynamic associated with the dynamic describing the basic process). My method replaces advantageously Pontryagin, which is quickly insoluble by the non-linearity or by the presence of constraints. In addition, I solved for all initial conditions, and do not need to solve a differential system, when it is possible, for each set of initial conditions. Application to the theory of the economy of the life cycle: calculation of the maximum viable utility an agent, who must decide between consumption, fertility, and savings during his lifetime. In addition, the dynamic is hybrid (which is a topical issue in control theory): controls are continuous in consumption, savings, and

[70] N. Bonneuil (2006) "Computing the Viability Kernel in Large State Dimension," *Journal of Mathematical Analysis and Applications* 323(2), 1444-1454.

[71] N. Bonneuil (2012) "Maximum under continuous-discrete-time dynamic with target and viability constraints," *Optimization* 61(8) 901-913.

financial performance, and discrete for the number and date of birth of children. Thanks to to a variant of my viability algorithm, adapted to large state dimension and discrete continuous controls, I calculate when it is optimal and viable to have children, taking into account the (dynamic) consumption of children and parents. I complete my article in *Journal of Mathematical Economics* 2008 on the life cycle and timing of fertility, where I developed the economic theory of the viable life cycle.

4.5 Viable Nash equilibria

As I mentioned, I have developed a powerful method to compute all Nash viable (the trajectories remain in a given closed set) equilibria (do the best possible considering that the other protagonists do the same), in large state dimension. Thanks to my theorem on obtaining optima as the boundary of a capture-viability kernel for an augmented dynamic, computation allows me to know all the optima (which is rarely the case by the usual procedures, which require solving differential equations for each initial condition). This set constitutes a manifold in the state space. The intersection of the manifolds obtained for each of the protagonists is exactly equal to the set of Nash equilibria.

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4.6 History and Theory of Time

I explained this new perspective of time I have developed in the study of populations,[72] by situating it in relation to chaos theory imported from physics and in relation to the narrative mode claimed by micro-history in particular. Instead of seeing a story as unfolding along a trajectory, I argued for set-valued analysis and differential inclusions in the social sciences, as being more adapted to human time, because the direction than the system can take at any time is extended to all admissible directions, which may come from a repertoire of desired actions. The challenge is often to move towards an objective (survival, conquer or maintain power, avoid poverty, achieve a successful fishing campaign, maintaining diversity,...) despite uncontrollable disruptions. I also discussed the contribution

[72] N. Bonneuil (2001) "History, Differential inclusions, and Narrative" " *History and Theory* Theme issue 40 'Agency after Postmodernism', Wesleyan University, 101-115.

of probability theory and dynamic game theory to History.[73,74,75]

I put philosophy of history and mathematics of time in perspective,[76] in particular by discussing the presence of properties such as continuity or connectivity in the narrative work. I also compared the contribution of probabilities to that of contingent dynamics (by differential inclusion) in the induction issue.[77] I have shed light on the asymmetry between past and future in the computation of sets of attainable states: the knowledge of subsets of the state space through which the system must go through in the past, more rarely in the future, imposes a restriction on the whole set of attainable states, and therefore the procedure is epistemic; towards the future, multiplicity of possible stories created by the possible and undetermined variation of a finite number of controls is intrinsic in the dynamics and therefore reflects an ontic understanding of future trajectories.

The "long march of the Church," from Augustine (5th century) to the Gregorian reform (1127), has finally succeeded in redefining the relationship between violence, the family, and God's ministers, and therefore between the sexes. I have proposed different interpretations of this founding event of Western civilization, by identifying Nash equilibria and saddle points in the space of emotions, by showing how courtly love possesses the characteristics of evolutionary innovations (in the sense of innovations at the molecular level), by identifying the mutation of relationships between the sexes and with God as morphological transformations of capture basins and sets of viability.[78]

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The time of men and women requires specific mathematics

The great challenge of demography is to think about time men and women, whether in the old demographic regime, where high irregularities of fertility over time reflect the maintenance or the extinction of communities subject to severe conditions of mortality, during

[73] N. Bonneuil (2004) "Repertoires, Frequentism, and Predictability," *History and Theory* 43(1), 117-123.

[74] N. Bonneuil (2005) "History and Dynamics: marriage or *misalliance*?," *History and Theory* 44(2), 265-270.

[75] N. Bonneuil (2008) "Do historians make the best futurists?," *History and Theory* 48, 98-104.

[76] N. Bonneuil (2010) "The mathematics of time in history," *History and Theory* 49, 27-45.

[77] N. Bonneuil (2013) "Viability, probabilities, induction," *Tracés*, 24, 71-84.

[78] N. Bonneuil, (2016) "Arrival of courtly love: moving in the emotional space," *History and Theory* 55(2), 253-269.

the transition, where the rate of change goes beyond simply adjusting to a changing environment, or during the baby boom, when fertility temporarily stabilizes after shifts. The causalities that one would think would be of common sense are misleading. Demography, observational science and not of experimentation, probably needs to appropriate a mathematical of original social and historical time, and to go beyond the notions of time often inherited from the 19th century physics taught at school. Instead of extending trajectories or imagining scenarios, one may wonder what actions or behaviors would either perpetuate or end a system that has been established over time.

I have brought mathematics closer to time and econometrics closer to demography. The fixed framework of stable populations or Markovian processes that dominate demography do not tell us about the people's time. Simulations, although very popular, are powerless to account for state spaces, even of dimension two, where controls vary continuously. Many models are based on conjecture, where the specified interaction tell a certain story, but rarely result from phenomenological considerations. The survival or failure of a system is an interesting principle in the social or natural sciences. The mathematics of viability makes it possible to translate these principles as faithfully as possible, and to confront the constraints to deduce what these interactions should be for the system to continue or have had to be for it to take the form we know it today. Rather than forecasting, these correspondences give the "map of the future": how the actors will have to react in a given situation if the system is to continue. I therefore propose a shift from our traditional concepts of stable population, simulation, forecasts, probabilities, equilibria, that I have also helped to bring to life in demography through several articles and two books, towards a conceptual framework and integrated mathematics, allowing us to think about uncertainty, the human, and the transient, replacing History and the mathematics of time at the heart of demographic transformations.